OPTIMIZATION OF THE AIR APPORTIONMENT IN A TAC THUNDER SCENARIO USING RESPONSE SURFACE METHODOLOGY

THESIS

Steven Lee Forsythe, Captain, USAF

AFIT/GST/ENS/94M-02

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DEPARTMENT OF THE AIR FORCE
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OPTIMIZATION OF THE AIR APPORTIONMENT IN A TAC THUNDER SCENARIO USING RESPONSE SURFACE METHODOLOGY

THESIS

Presented to the Faculty of the Graduate School of Engineering

of the Air Force Institute of Technology

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In Partial Fulfillment of the

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Master of Science in Operations Research

Steven L. Forsythe, B.S.

Captain, USAF

MARCH, 1994

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Steven Lee Forsythe

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LIST OF ABBREVIATIONS

AAPk - air-to-air probability of kill
A/C - aircraft
AD - air defense
AFM - Air Force Manual
AIRESC - air-to-air escort
BAI - battlefield air interdiction
BARCAP- barrier combat air patrol
CJAM - close-in air defense jamming
CSUP - close-in lethal air defense suppression
DCA- defensive counter-air
DSEAD - direct lethal air defense suppression
EJAM - air defense jamming escort
ESUP - lethal air defense suppression escort
FLOT - forward line of own troops
FSWP - fighter sweep
INT - air interdiction
MOE - measure of effectiveness
MSE - mean square error
OCA - offensive counter-air
ODCA - over-FLOT defensive counter-air
Pk - probability of kill
RECCE - reconnaissance
RES - reserve
RSM - response surface methodology
SAM - surface-to-air missile
SAPk - surface-to-air probability of kill
SJAM - stand-off air defense jamming
SSUP - stand-off lethal air defense suppression
STI - strategic targets interdiction

ABSTRACT

TAC THUNDER is the Air Force's premier campaign level model. Perhaps the most important input to TAC THUNDER is the user-defined apportionment of available aircraft. This apportionment defines the percentage of each type of aircraft that are assigned to specific missions. An optimal apportionment represents the best use of aircraft available to the theater commander. Campaign outcomes using an optimal allocation are a powerful tool for demonstrating that no superior non-material solution exists as far as aircraft employment is concerned. That is, an optimal allocation addresses Congress's concern that US military commanders fully exploit current weapons systems before acquiring replacement systems.

To effectively compare different sets of available aircraft, it is necessary to find the apportionment that maximizes the effectiveness of each set of aircraft. This research effort uses an unclassified scenario to show how an analyst can use response surface methodology (RSM) techniques to estimate the relationship between aircraft apportionment and campaign outcomes.

The optimal apportionment for one phase of a hypothetical conflict was found by using a steepest-gradient search of the constrained response surface. The results are illustrated by illuminating charts showing the various relationships between the number of aircraft in the theater and measures of effectiveness such as FLOT movement and attrition. The results show close air support and interdiction missions to be highly effective in this phase of the scenario. The effect of the enemy's air-to-air probability of kill (AAPk) and surface-to-air probability of kill (SAPk) on the MOEs provides insight into the robustness of the solution in terms of these operational uncertainty parameters. This type of sensitivity analysis also provides useful information on how the TAC THUNDER algorithms interact to simulate the combat environment.

I. Introduction

> It is absolutely necessary to study new systems in the context of the environment in which they will participate. We must measure the interdependencies that are becoming a bigger and bigger part of a system's contribution to the overall campaign (Smith, 1992;13)

In this era of declining defense expenditures and increasingly expensive research and development programs, the U.S. must focus its R&D dollars on those programs that add the most capability per tax dollar spent. Computer simulations of future battlefields provide an accepted framework for comparing alternative weapon systems. In evaluating such alternatives, the effectiveness of each weapon system must include its own capabilities, as well as its contribution to the combat effectiveness of the overall force.

The Aeronautical Systems Center (ASC) uses a computer simulation called TAC THUNDER to simulate a theater-level conflict. ASC uses TAC THUNDER to evaluate the effectiveness of various aircraft designs and modifications. By simulating a conflict repeatedly, analysts can measure the effect of changing the number and type of aircraft on the outcome of the hypothetical war. From a force analysis point of view, the results of a simulated war would ideally depend on only the characteristics of the forces involved. Not surprisingly, the manner in which forces are employed has a tremendous impact on the outcome of the war.

Current acquisition regulations recognize that a weapon system's effectiveness depends on force employment doctrine. The justification for a new weapon system must show that the deficiency the system is designed to eliminate cannot be eliminated by using current systems more effectively. Using current systems more effectively is referred to as a "non-material solution". The non-material solution issue could be resolved if an optimal force employment strategy for both sides could be found. However, theater-level combat models are generally very large and complex. To simultaneously optimize the use of ground, sea, and air forces on both sides of a large simulated conflict is computationally intractable. ASC/XREC, the campaign analysis office for ASC, sponsored this thesis in an effort to develop a method to determine if non-material solution alternatives are available for Air Force weapon system proposals.

II. Background

Introduction

This chapter provides a background on the material used in this thesis. It is intended to familiarize the reader with the diverse topics covered and provide references, via the bibliography, for those readers interested in additional in-depth information. The major topics covered are the air campaign, air apportionment, TAC THUNDER, and response surface methodology (RSM). The air campaign is the primary focus of the computer simulation TAC THUNDER. One of the major user inputs to TAC THUNDER is the apportionment of air power to various mission classes. RSM is the set of techniques used to investigate the functional relationship between the air apportionment and the outcomes of the simulated campaign. Applying optimization methods to this functional relationship can identify the best air apportionment for a given scenario and measure of effectiveness.

Air Campaign

AFM 11-1 defines the *air campaign* as "A connected series of operations conducted by air forces to achieve joint force objectives within a given time and area of operations" (AFM 1-1, 1992). In order to execute this series of operations, the air commander must decide where to focus his efforts. He must apportion his aircraft between the various missions they can perform. For example, air superiority aircraft can be used offensively or defensively. Similarly, aircraft that deliver munitions can do so to a variety of target classes such as front line troops, reserve troops, bridges, and airfields. The percentage of available aircraft assigned to perform each of these missions is referred to as the aircraft apportionment.

It is vitally important that the commander use his resources most effectively to accomplish his objectives. Technical superiority is not sufficient to guarantee victory (Warden, 1989).

Air Apportionment

"Allocation strategy is the most powerful driver of all the factors which determine the outcomes of campaigns and wars" (Smith, 1992;3). Air forces are allocated through the air apportionment. It assigns a percentage of available aircraft to specific missions and can change from day to day.

The missions defined in TAC THUNDER (version 5.9) are:

- **Close air support (CAS)** - attack units engaged in direct fire combat.
- **Battlefield air interdiction (BAI)** - attack support and 2nd echelon units.
- **Air interdiction (INT)** - attack the enemy's logistic facilities, C^3 facilities, choke points, transshipment points, rear-area units, or supply trains.
- **Offensive counter-air (OCA)** - attack the enemy's capability to generate sorties. These missions exclusively attack air bases.
- **Strategic targets interdiction (STI)** - attack the enemy's strategic targets.
- **Fighter sweep (FSWP)** - an offensive air-to-air mission, attacks enemy aircraft that are operating on their own side of the FLOT during a period when friendly ground attack aircraft are operating in the vicinity.
- **Defensive counter-air (DCA)** - aircraft sit on the ground and wait for the enemy's OCA or INT attacks. A DCA aircraft flies only in response to an attack. If it takes off and fails to intercept the enemy, it acts exactly like a BARCAP aircraft.
- **Barrier combat air patrol (BARCAP)** - aircraft, while on station, patrol a designated area and attempt to intercept any enemy aircraft that attempt to pass.

- **Over-FLOT defensive counter-air (ODCA)** - aircraft sit on the ground and wait for a certain type and size enemy flight to take off. An ODCA aircraft then flies up to a user-defined distance into enemy territory to attack the flight.

- **Stand-off lethal air defense suppression (SSUP), stand-off air defense jamming (SJAM), close-in lethal air defense suppression (CSUP), and close-in air defense jamming (CJAM)** - aircraft orbit the battlefield and are available to support other missions that are threatened by enemy air defenses. The two primary differences between close-in and stand-off missions lie in the manner in which targets are assigned and generated and the aircraft orbit location with respect to the FLOT. As far as mission execution, there is no difference.

- **Direct lethal air defense suppression (DSEAD)** aircraft attack air defense sites within a corridor. Aircraft will attack the assigned air defense sites for a given amount of time before the corridor becomes active.

- **Reconnaissance (RECCE)** - collects intelligence about the enemy.

- **Lethal air defense suppression escort (ESUP)** - aircraft accompany missions to provide lethal suppression of enemy air defenses.

- **Air defense jamming escort (EJAM)** - aircraft accompany mission to provide jamming suppression of enemy air defenses.

- **Air-to-air escort (AIRESC)** - aircraft accompany missions to provide air-to-air protection.

- **Reserve (RES)** - aircraft being held for possible future use.

Not every scenario requires every mission type. For example, strategic target interdiction (STI) is used only when there are strategic targets in the theater. These targets are usually more political in nature and do not directly impact the capabilities of the forces fighting. Some scenarios require that 100% of the aircraft be assigned to non-reserve missions.

TAC THUNDER

TAC THUNDER is a piston driven, theater-level combat model. It uses an aggregated, deterministic ground war and a detailed, stochastic air war to simulate a theater-level conflict (TAC THUNDER Analyst's Manual, 1992). It models ground units at the regiment and division level, consistent with the Army's FORCEM model. Aircraft sorties are modeled individually.

By simulating the same theater-level campaign under different conditions, such as the addition of new equipment (or the substitution of one set of equipment for another), TAC THUNDER provides a means to assess the effect of those changes on the campaign objectives. The advantage of a theater-level model over smaller, higher fidelity models is that it measures the impact of each alternative in terms of the overall theater campaign.

Each TAC THUNDER scenario requires over sixty data files to define the size, shape, and geography of the theater as well as the forces and objectives of both sides. The model uses this input data to assign aircraft to specific targets on each day of the war. Most of the data files are simply descriptive, but the apportionment of available aircraft into TAC THUNDER-defined missions is a critical input. Since the apportionment determines the allocation of air resources, an unacceptable campaign outcome might be due to an inappropriate apportionment rather than to inadequate aircraft capabilities. To ensure that the outcomes reflect the capabilities of the aircraft, the apportionment file must assign the right aircraft to the right missions.

In TAC THUNDER, aircraft are assigned to *mission classes*; a percentage of each mission class is then apportioned to specific missions. For example, F-15 and F-14 aircraft are grouped together to form the air superiority mission class. The air apportionment assigns a percentage of air superiority aircraft to air escort, fighter-sweep, and barcap missions. Other mission classes include multi-role, deep strike, and ground support.

TAC THUNDER is written in SIMSCRIPT II.5. SIMSCRIPT is a structured simulation language which is designed to be nearly self-documenting. TAC THUNDER requires 500 megabytes of hard drive space and can be run on any machine that runs SIMSCRIPT II.5. A SUN workstation or DEC station is required to support the terminal graphics used in TAC THUNDER's situation map and grapher.

Response Surface Methodology (RSM)

RSM is a structured set of techniques for empirical model building and model exploitation (Box and Draper, 1987;1). These techniques include design of experiments, least squares regression, and optimization. Several good texts exist to assist readers who wish more information on RSM (Box and Draper, 1987), design of experiments (Schmidt, 1991), or applied regression (Draper and Smith, 1981).

RSM seeks to relate a *response* or *output variable* to the values of a number of *predictors* or *input variables.* The prediction of the response to various input variables is called a *response surface*. Since a simulation is a model itself, the response surface of a simulation is sometimes referred to as a *metamodel* (a model of a model).

Empirical models attempt to explain how the inputs relate to the output of a process. Empirical model building estimates the response in an area of immediate interest. The true response function is unknown but it is assumed that it can be locally approximated by a polynomial or some other type of function. This polynomial can be conceptualized as an "analytic French curve." It approximates the true response function over a small experimental region.

If the air apportionment can be reduced to a reasonable number of factors, then standard RSM techniques can convert a set of simulation runs conducted in accordance with an experimental design into a metamodel of the simulation.

Sequential experimentation is a structured method of scientific investigation. This research used sequential experimentation as the framework to guide the application of RSM techniques. Sequential experimentation consists of an iterative cycle of investigation: CONJECTURE, DESIGN, EXPERIMENT, and ANALYSIS. Figure 2.1 illustrates this cycle. *Conjecture* is the hypothesis that the analyst wants to test. *Design* is the synthesis of a suitable experiment to test, estimate, and develop a current conjectured hypothesis. *Analysis* is the treatment of the experimental results, leading either to verification of a postulated model and the working out of its consequences, or to the forming of a new or modified conjecture (Draper, N. R. and H. Smith;8).

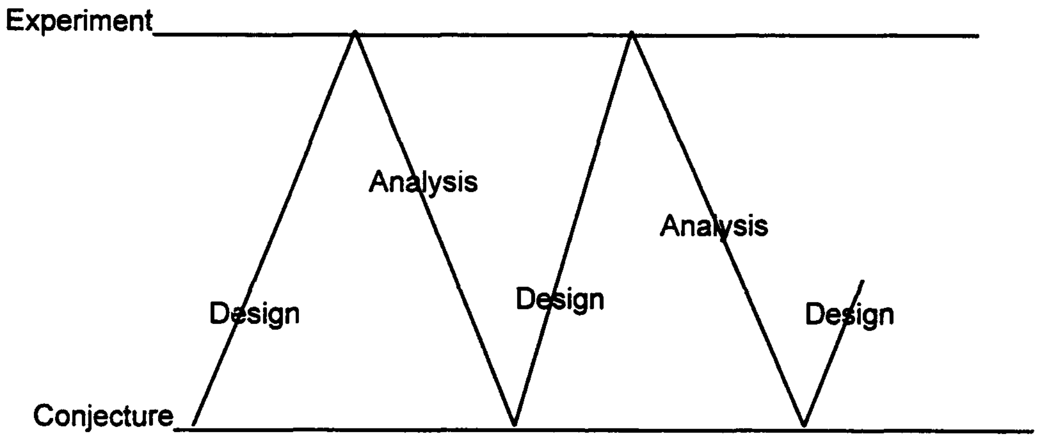


Figure 2.1 The iterative nature of experimentation. (from Empirical Model-Building and Response Surfaces by Box and Draper)

The conjectures for a particular experiment depend on the experimenter's understanding of the process under study. In the first stage of studying a process, the conjectures focus on identifying WHICH variables are important. After identifying WHICH are the important variables, the conjectures focus on identifying HOW the response behavior relates to the important input variables. This stage is often called empirical model building since the product is an empirical relationship between inputs and outputs. Finally, the conjectures focus on WHY the process responds the way it does. This stage is often called mechanistic model building because the product is an understanding of the underlying mechanisms of the process.

The conjecture establishes the information required by a particular experiment. The design should be of sufficient resolution to verify or disprove the conjecture. A design is said to alias or confound two factors if the design does not allow them to be distinguished from one another.

A design is said to be of resolution k if all n^{th} order terms are not aliased with any terms lower than order k-n, where $n < k$. For example, in an R_{III} design (k=3), the 1^{st} order (n=1) terms in a resolution design are aliased with 2^{nd} order or higher terms (k-n=2). Thus, R_{III} designs are considered 1^{st} order designs because no 1^{st} order terms (main effects) are aliased with one another. Likewise, R_{IV} designs are considered 1^{st} order designs. Although 1^{st} order terms in an R_{IV} design are not aliased with any terms lower than 3^{rd} order, the 2^{nd} order terms are still aliased with one another. Resolution five designs are ideal for 2^{nd} order models since the 1^{st} order terms are not aliased with any terms lower than 4^{th} order, and the 2^{nd} order terms are "free of" confounding with any terms lower than 3^{rd} order. Screening designs most often use R_{III} or R_{IV} designs to determine WHICH of many variables under consideration are most important. Designs of R_V or higher are generally used to establish HOW the input variables affect the response(s). Determining WHY the process behaves as it does entails a sequential process of scientific inquiry intended to establish a theoretical basis for the observed phenomena. The experimental designs used to address the fundamental WHICH, HOW, and WHY questions provide the basis for empirical and mechanistic model building.(Box and Draper, 1987; 11-14)

The designs in this study are all two level designs, where each input variable is at either a high (+1) or low (-1) setting. These designs are used to estimate models with terms consisting of linear combinations of the input variables and their products. Appendix C provides the specific designs used in this thesis.

Analyzing an experiment utilizes the RSM techniques of multivariate regression to analyze the data and identify the one best model (or metamodel). Several statistical "goodness of fit" measures exist to assist an analyst in making this determination. This information is frequently presented in an ANOVA table which summarizes the information relating the least squares fit of the model to the data. The R^2 value represents a measure of how much of the structure of the data can be explained by the model. An R^2 value of one means the model fits the data perfectly. As the number of terms in the model increases, the R^2 value goes up (or possibly stays the same). The adjusted R^2 value takes into account the number of degrees of freedom used by the model. Each term in the model uses one degree of freedom. The total available number of degrees of freedom is equal to the number of data points.

The Mean Square Error (MSE) is an estimate of the variance of the process, if the model is assumed to be correct. If the model is not correct, MSE is the sum of the variance due to the process and the lack of fit.

The partial F statistic is the ratio of the sum of squares to the MSE for each term in a specific model. It represents the probability that the coefficient associated with each term is actually zero.

Mallow's CP statistic requires an estimate of the variance of the process under study. If the number of terms in a model is much less than the CP statistic, then that model does not mirror the process under study well.

Each of these statistical measures should be considered when analyzing possible models. No single measure will identify the "best" model. Simplicity of the model is also an important feature. A simple model may be more useful than a more complex model that fits the data marginally better. For example, Newtonian equations of motion are less accurate than relativistic equations of motion, but when the velocities involved are much less than the speed of light, the simpler Newtonian models are usually used. At these velocities the differences between the output of the two models is negligible.

RSM has often been used to optimize a specific simulation output as a function of the inputs (Harvey, Bauer, and Litko, 1993). Analysts use simulations to study complex situations with complex input-to-output responses. Analysts use RSM to clarify the relationship between the simulations' input variables and the simulations' output.

Once the relationship between the inputs and outputs is estimated as a metamodel, the metamodel is optimized using a variety of techniques (Garrison, 1992). If the metamodel is a linear combination of variables, the magnitude and sign of the coefficients associated with the variables indicate the contribution each variable makes to the objective. It is important to note that RSM techniques usually scale variables to +1 and -1 values (or similar set of scaled values). To optimize the simulation output, the metamodel estimated using the scaled variables must be transformed into the original variables. Such a transformation can change the relative values of the coefficients.

If the model is a complex, nonlinear model, then steepest ascent search techniques are used to find the optimal solutions. Steepest ascent searches are commonly used algorithms which attempt to locate the maximum (or minimum) values of nonlinear functions. A commercial package known as GINO was used to optimize the complex metamodels. GINO supports the optimization of nonlinear objective functions subject to constraints.

Conclusion

TAC THUNDER is a complex computer simulation of a theater-level conflict. In order to use it to estimate the effectiveness of a specific set of aircraft, the analyst must use these forces optimally relative to the objectives of the campaign. RSM can be used to identify and understand the relationships that exist between simulation input (aircraft apportionment) and simulation output (the results of the conflict). Additional sources of information on the techniques and methods used in this research endeavor are listed in the bibliography. Chapter three contains more information on the specific TAC THUNDER experiments conducted as part of this thesis and the resulting metamodel.

III. Baseline Experiment

Introduction

This thesis used a sequential experimentation approach to optimize the use of aircraft within a single TAC THUNDER scenario. Each phase in the investigation provided information and insight used to refine and direct subsequent phases throughout the research effort. The general iterative, experimental approach consists of **conjecture**, **design**, **experimentation**, and **analysis**. This chapter presents the details of the conjecture, design, experimentation, and analysis efforts associated with the baseline experiment.

Conjecture

Theater-level campaign analyses, using *proper analytical treatment,* provide the only meaningful way of estimating the contributions of systems that perform disparate functions in wartime (Goodson, 1993;1). General Goodson defines *proper analytical treatment* as mathematical search algorithms that find the best use of force. Comparisons between force options are only meaningful if all the force options have been used to their maximum effectiveness. Such comparisons are difficult because theater-level conflicts are large and complex by nature, and the relationships between force employment and battle outcomes are not known ahead of time. For combat models, like TAC THUNDER, that do not optimize the apportionment of forces, an analyst must input the best apportionment for each resource under study to insure that the outcomes of each conflict reflect the capabilities of the resources available. This research was based on the premise that it is possible to estimate the relationship between the air apportionment and the outcomes of a theater-level scenario.

The objective of this thesis is to optimize the air apportionment with respect to each of three measures of effectiveness (MOE): The movement of the forward line of own troops (FLOT), the strength of friendly ground forces when the opponent's advance is halted, and the attrition of aircraft.

This research used an unclassified scenario based on Desert Storm. The original scenario provided by CACI had to be modified. The original scenario modeled the ground war phase of Desert Storm. That scenario was too unbalanced to show a strong relationship between the best use of aircraft and the campaign outcomes. The forces in the scenario were changed to reflect a strong Iraqi attack from Kuwait into Saudi Arabia not long after they had invaded Kuwait. The coalition ground forces were outnumbered and significant additional ground forces were 30 days away. The objective of the coalition commander was to stop the Iraqi advance while minimizing the loss of coalition air and ground power. It was presumed that once the FLOT was stopped, this objective had been met. Additional forces would arrive later and eventually attack, but the best use of forces in those situations is not likely to be the same as during this initial advance. The optimal air apportionment sought related only to halting the opponent's advance. The optimal apportionment for subsequent operations was not part of this research effort. Additional details of this scenario are available in Appendix A.

Design

There are 12 types of aircraft and 17 missions defined for this scenario. A 30 day war would require 12x17x30 or 6120 variables to represent each aircraft/mission/day combination. If the relationship between the inputs and the outputs were known and linear, a linear program could optimize the air apportionment. Unfortunately, these relationships are not known, so some other approach is required.

RSM was chosen as the best approach, given the nature of the problem, but it required reducing the number of variables to a manageable level. The number of variables impacts the number of simulation runs required to estimate the relationship between the air apportionment and battle outcomes. Simulating ten interations of a 30 day conflict took about six hours on a Sun workstation. The computational time required to do hundreds of runs exceeds the time available for many study efforts including this thesis effort. Studies accomplished after a decision is made tend to be of little value to a decision maker.

Therefore, the following assumptions were made to simplify the problem:

- The opponent's air apportionment was fixed. It represented our best estimate of his air plan. The simulation results applied only to this particular strategy.
- Only the "best" aircraft for each mission were included in this simulation: F-15's for air superiority missions, F-15E's for INT and BAI, F111's for OCA, A-10's for CAS. It is assumed that if other aircraft were assigned to these missions, the results would not be better (they may be the same or worse).
- A constant air apportionment is acceptable for the first 30 days of this scenario since the situation remains relatively constant.

The input variables were the number of aircraft assigned to each of the missions listed on pages 2-2 through 2-4. Not all of these missions necessarily contributed significantly in this scenario. Therefore, a screening design was chosen to identify the missions that were most significant. A second experiment could be run later using only these significant variables to develop a higher order model.

A 2^{9-5}_{III} fractional factorial design was chosen for the initial experiment. It would permit estimation of the main effects and could be supplemented with an additional fraction, if necessary. The 2^{9-5} indicates the experiment consists of 16 (2^4) design points, and the "III" indicates that it is a resolution three design. Statgraphics software was used to help design the experiment and analyze the data. Statgraphics generated a design matrix that showed the value of each input variable for each of the 16 runs. These values were coded as +1 or - 1. Each mission had a maximum and minimum number of aircraft that were assigned to fly that mission on the first day of the war. These values define the boundaries of the input variables. The estimated response surface was confined to points bounded by these minimum and maximum values.

Experimentation

This section describes the implementation of the fractional factorial design. Important decisions on the implementation had to be made. These decisions included how many times TAC THUNDER would be run at a single design point, and what should be the maximum and minimum number of aircraft assigned to each mission.

The output of a single TAC THUNDER run is a random variable. Additional runs with identical inputs allow a more precise estimation of the mean response of TAC THUNDER to those inputs. Additional runs also require more time and computer resources. For this experiment, it was decided to repeat each design point ten times. Ten repetitions reduced the uncertainty bounds on mean result at each data point. Additional runs did not improve the uncertainty bounds enough to justify allocating the additional resources.

The maximum and minimum number of aircraft assigned to each mission defined the variable space for this experiment. The lower bound for each mission class was selected to be the fewest number of aircraft a commander might realistically assign. For example, a commander would never voluntarily have no defensive air capability. The maximum value was selected as the largest number of aircraft a commander might assign and still not encounter significant diminishing returns. Clearly, if enough aircraft are currently assigned to destroy every interdiction target in the theater, the value of additional interdiction sorties is very small. Appendix A contains the maximum and minimum values of aircraft assigned to each mission class.

The TAC THUNDER files that contain information on each squadron were changed to reflect the appropriate number of each type of aircraft for each design point. The TAC THUNDER air apportionment files, however, assign a percentage of each mission class to specific missions rather than specific numbers. In the case of CAS and BAI, this problem was solved by assigning 100% of the aircraft of a specific class (ground support and multi-role) to those missions. The multi-role class was changed to perform as if they were all F15E's. The available deep strike aircraft were divided between OCA and INT missions. The air superiority class was divided between DCA, FSWP, and AIRESC missions. The percentages of each mission class assigned in the air apportionment file were adjusted so that the number initially assigned to each mission reflected the high or low values associated with each mission for that run. For example, if DCA, FSWP, and AIRESC were all at their max or min values (72 or 12) then the air superiority mission class would be assigned 33% to DCA and AIRESC missions and 34% to the FSWP mission. If the design point required 72 FSWP and 12 DCA and AIRESC missions, the air superiority mission class would be divided as follows: 75% to FSWP, and 12.5 % to DCA and AIRESC respectively.

TAC THUNDER has a graphics post processor that creates user-defined macros to display the results of each run. Due to the tremendous amount of data associated with each run, a script was created that stored the information for the macros and then deleted everything else before starting the next repetition of that run. This was a compromise between the desire for large statistical samples and the limitations on time. Each iteration took approximately 30 minutes to run on a Sparc II workstation. Three separate runs were done simultaneously on one machine. Approximately 18 hours later, those runs were finished and new set of runs was started. Occasionally the TAC THUNDER program would generate an error that would stop a batch run and require restarting the run.

The output for each set of runs was condensed into one file and printed out four to a page, double sided. It was necessary to look at the FLOT movement to determine which day the FLOT movement reached its final position. The air and ground attrition numbers used were those associated with that day. The 10 iterations of runs were averaged to yield the response inputs for statgraphics. The design and the results of the initial screening experiment are listed in Appendix C.

Analysis

Estimating the Response Surfaces: Initially, the aircraft losses were expressed as a percentage of the initial inventory available. This response was unsuitable because the variance of the response was a function of the value of the response. One of the fundamental assumptions of regression is that the variance of the response is constant throughout the design region. Expressing the losses in terms of aircraft lost essentially eliminated the problem of heteroschodasticity.

The original objective of the screening design was to identify WHICH of the variables were important. However, high quality metamodels were estimated with the data from the screening design experiment, rendering subsequent higher resolution designs unnecessary.

The MOEs were strongly correlated to the experimental setting of the CAS and INT missions. In Figures 3.1 through 3.3, a lowercase "c" or "i" indicates a low level for CAS or INT, while a capital "C" or "I" indicates a high level. Larger FLOT values represent a deeper Iraqi penetration into Saudi Arabia.

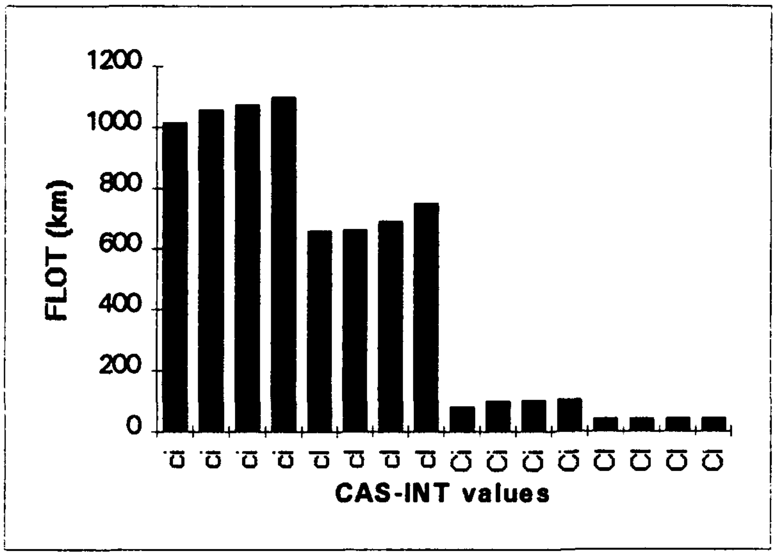


Figure 3.1 FLOT results grouped by CAS and INT

Notice the overwhelming effect of CAS missions in Figure 3.1. The higher proportion of CAS missions significantly reduced the Iraqi advance. INT missions also contributed to significantly reducing the FLOT movement.

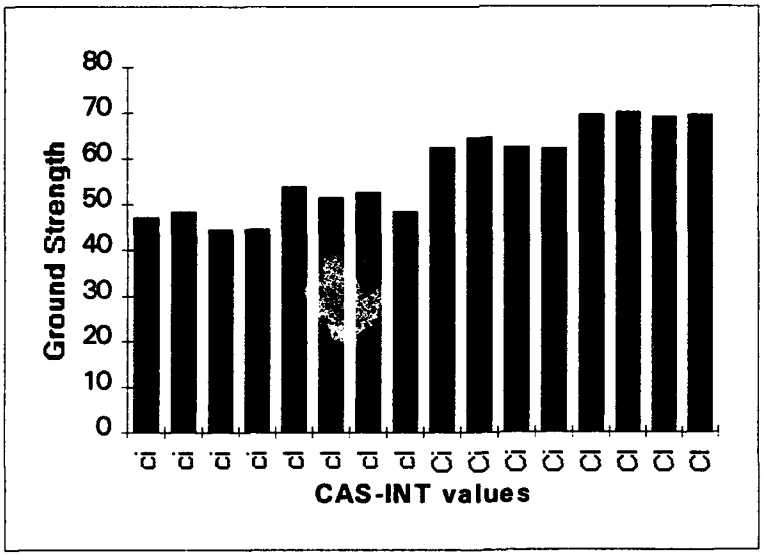


Figure 3.2 Ground Strength results grouped by CAS and INT

In Figure 3.2, CAS and INT missions significantly influence the response of the MOE (ground strength). Higher levels of CAS and INT generally led to higher levels of remaining allied ground strength.

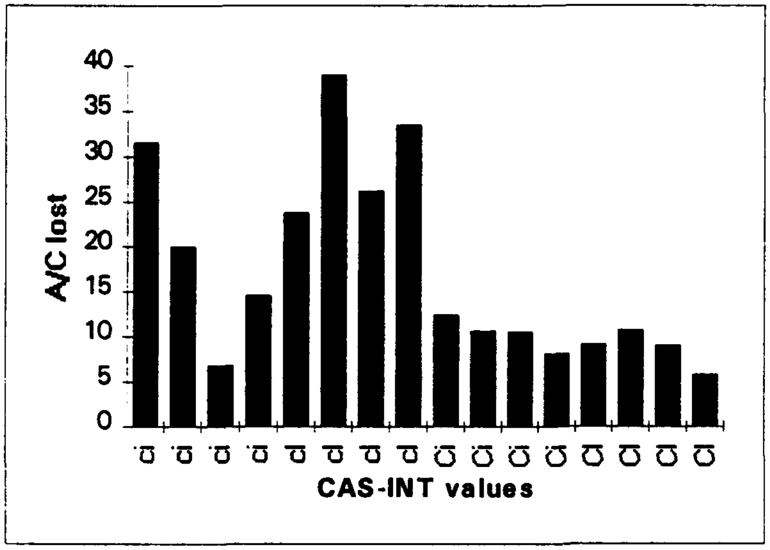


Figure 3.3 A/C attrition grouped by CAS and INT

Figure 3.3 shows one of the most unexpected results of this study. The fewest number of aircraft tended to be lost when CAS missions were at the high level. A closer examination of the data revealed that the high level of CAS generally resulted in objectives being met more expeditiously. The shorter conflict reduced the total number of aircraft sorties. Fewer sorties exposed to the threat resulted in fewer aircraft lost. Another possible explanation is that the CAS missions reduced the intrinsic air defenses of the Iraqi units. These intrinsic air defenses consist of portable surface-to-air missiles (SAM's), mobile SAM's, and gun systems used in air defense.

The rest of this chapter presents the metamodels estimated from the screening design data. The ANOVA table associated with metamodels as well as diagnostic plots are provided to assist the reader in understanding how the various metamodels were estimated. The estimation of the FLOT model includes a description of how examination of the residuals led to the inclusion of a nonlinear term in the metamodel.

The ANOVA for the purely linear FLOT model is given in Table 3.1.

Table 3.1 ANOVA of FLOT model (Linear)

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
CAS	2594274	1	2594274	309	.0000
INT	179289	1	179289	21.3	.0000
Total Error	109194	13	8400		
Total Corrected	2882757	15			

$R^2 = 0.962$ R^2 (adjusted for degrees of freedom) = 0.956

While the adjusted R^2 of the linear FLOT model was high, the distinct U-shape of the residual plot in Figure 3.4 indicates the need for a higher order term in the model. Ideally, the residuals reflect random deviations from the model that are only "noise" terms and are distributed normally about a mean of zero. A distinct pattern in the plot of the residuals, such as is found in Figure 3.4, indicates the presence of an unaccounted effect. Adding the CASxINT interaction term to the model removed all discernible patterns from the residual plot.

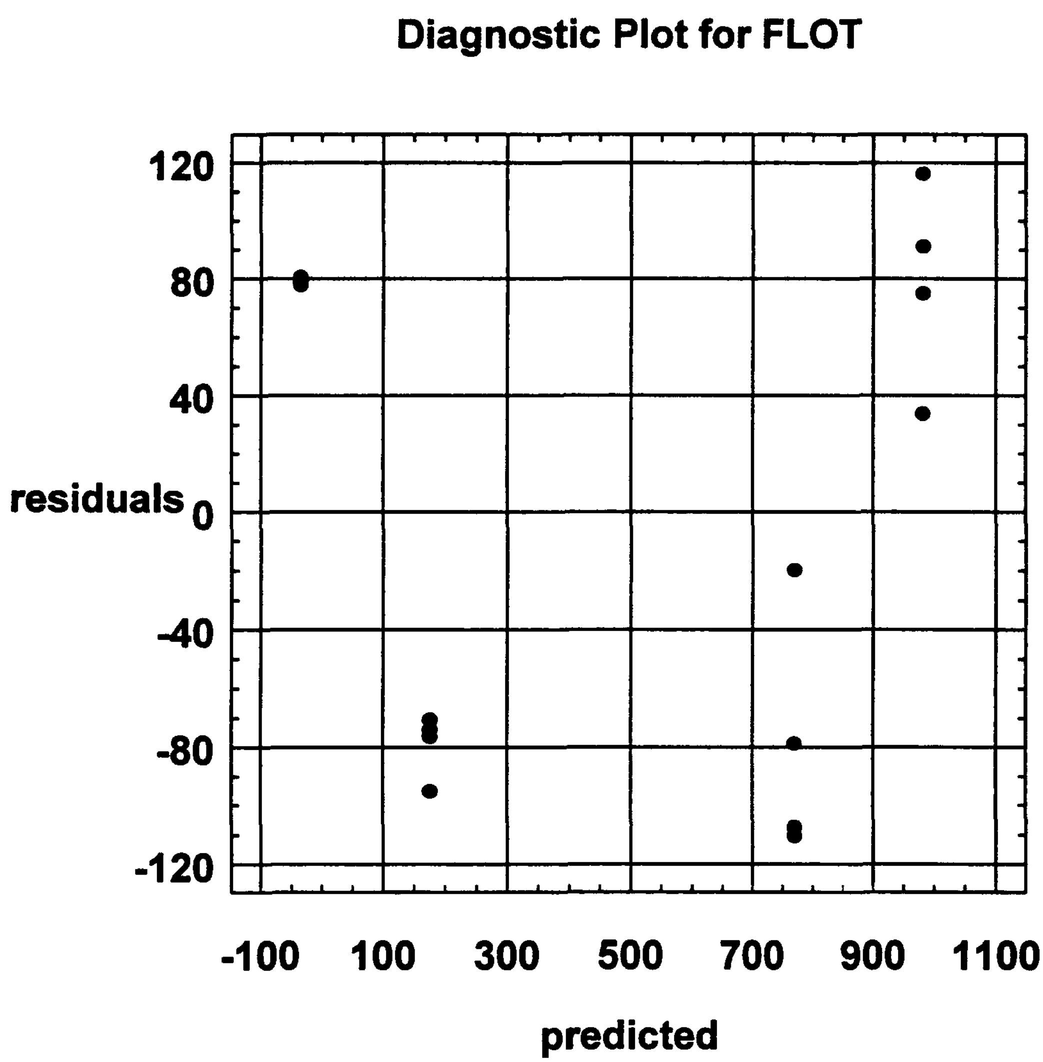


Figure 3.4 Plot of residuals for FLOT

The final estimated FLOT model is:

$$FLOT = 1061 - 4.460 * CAS - 3.852 * INT + 0.01525 * CAS * INT \quad \text{(Eq 3-1)}$$

Where:

FLOT is the total advance of the enemy in kilometers.

CAS is the number of close air support sorties assigned.

INT is the number of interdiction sorties assigned.

The ANOVA for the FLOT model is given in TABLE 3.2.

Table 3.2 ANOVA for the FLOT model (nonlinear)

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
CAS	2594274	1	2594274	3362	.0000
INT	179289	1	179289	232	.0000
CAS*INT	99935	1	99935	130	.0000
Total Error	9259	12	772		
Total Corrected	2882757	15			

$R^2 = 0.9968$ $\qquad$ R^2 (adjusted for degrees of freedom) = 0.9960

The adjusted R^2 of .99 represents a nearly perfect fit. The F-test of the CASxINT term is highly significant, and the new equation models the response almost perfectly. Equation 3-1 matches the data representing the FLOT movement nearly exactly.

The estimated model for the ground strength remaining at the end of the first phase of the campaign is:

$$GrdStr = 45.847 + 0.080444 * CAS + 0.063021 * INT \qquad \text{(Eq 3-2)}$$

where

Grd Str is the combat firepower of the friendly ground forces when the FLOT is halted.

Table 3.3 gives the ANOVA for the ground strength model defined in Equation 3-2.

Table 3.3 ANOVA for ground strength model

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
CAS	1207.6	1	1207.6	462	.0000
INT	146.4	1	146.4	56	.0000
Total Error	34.0	13	2.6127		
Total Corrected	1388	15			

$R^2 = 0.9755$ $\qquad$ R^2 (adjusted for degrees of freedom) = 0.9718

An adjusted R^2 of .97 represents an excellent fit. The residuals (which represent the distribution of the error term) appear to be distributed normally based on the residual plot and the normal probability plot of the residuals.

The estimated model for the aircraft attrition is:

$$\begin{aligned} AirAtr = {} & 7.66 + .160 * OCA + 0.0152 * AIRESC + 0.0477 * BARCAP \\ & + 0.1102 * FSWP - 0.0363 * CAS + 0.0475 * BAI \\ & - 0.001046 * OCA * INT + 0.1206 * INT - 0.00199 * AIRESC * FSWP \\ & - 0.00681 * CAS * INT \end{aligned} \qquad \text{(Eq 3-3)}$$

Where

AirAtr is the number of friendly aircraft lost.

OCA represents the number of OCA missions flown on the first day of the war.

AIRESC represents the number of AIRESC missions flown on the first day of the war.

BARCAP represents the number of BARCAP missions flown on the first day of the war.

FSWP represents the number of FSWP missions flown on the first day of the war.

BAI represents the number of BAI missions flown on the first day of the war.

Except for attrition, the input variables are constant.

Table 3.4 gives the ANOVA for the air attrition model defined in Equation 3-3.

Table 3.4 ANOVA for the air attrition model

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
OCA	141.0	1	141.0	161	.0001
AIRESC	139.8	1	139.8	159	.0001
BARCAP	32.8	1	32.8	37	.0017
FSWP	10.1	1	10.1	11.5	.0195
CAS	890	1	890	1014	.0000
INT	113	1	113	129	.0001
BAI	83.3	1	83.3	95	.0002
OCA*INT	29.4	1	29.4	34	.0022
AIRESC*FSWP	51.5	1	51.5	59	.0006
CAS*INT	200	1	200	227	.0000
Total Error	4.39	5	.87762		
Total Corrected	1694.2	15			

R^2= 0.99741 R^2 (adjusted for degrees of freedom) = 0.99223

Table 3.4 shows the fit for an 11 term model of the air attrition. The model for air attrition is an excellent fit to the data and each of the terms appears to be significant according to the partial F-test. Obviously air attrition is a complex process. The only concern with this model is the low degrees of freedom for the error term.

Optimization of the Response Surfaces: GINO, a steepest gradient search tool, was used to optimize the air attrition model. The other two models were of sufficient simplicity to be optimized analytically.

The FLOT model (Equation 3-1) was optimized by taking partial derivatives with respect to each of the variables and equating them to zero. The partial derivatives of the FLOT model are:

$$\frac{\partial FLOT}{\partial CAS} = -4.46 + .01525 * INT$$
$$\frac{\partial FLOT}{\partial INT} = -3.852 + .01525 * CAS \qquad \text{(Eq 3-4)}$$

The minimum estimated FLOT movement occurs for 253 CAS aircraft and 292 INT aircraft. Both of these values are outside the design region. The minimum estimated FLOT movement, therefore, occurs on the boundary of the design region. In this case, the minimum occurs for 216 CAS aircraft and 96 INT aircraft.

The ground strength metamodel (Equation 3-2) is a linear combination of CAS and INT missions. The ground strength is maximized when CAS aircraft and INT aircraft are maximized. Once again, the optimal result occurs for 216 CAS aircraft and 96 INT aircraft.

Complex nonlinear models such as the air attrition model are not as easy to analyze. GINO, a commercially available analysis software, was used to optimize the air attrition equation with constraints. The optimization problem was set up as follows:

Minimize: 7.664 + 0.1602*OCA + 0.01519*AIRESC + 0.0477*BARCAP +
0.1102*FSWP - 0.03473*CAS + 0.04752*BAI - 0.001046*OCA*INT +
0.1206*INT- 0.001993*AIRESC*FSWP - 0.000651*CAS*INT;

Subject to the following constraints:

OCA + AIRESC + BARCAP + FSWP + CAS + INT < xxx (xxx represents the maximum number of aircraft in the theater)

.076885*CAS + .063021* INT + 45.8 > yyy (yyy represents the minimum acceptable ground strength level. If yyy = 0 then this constraint is inactive.)

AIRESC - OCA - 4*INT < 0 (This constraint limits the number of escort missions to the maximum possible based on the number of escortable missions planned.)

OCA > 18 (The minimum number of OCA missions)

AIRESC > 12 (The minimum number of AIRESC missions)

FSWP > 12 (The minimum number of FSWP missions)

BARCAP > 12 (The minimum number of BARCAP missions)

BAI > 0 (The minimum number of BAI missions)

INT > 0 (The minimum number of INT missions)

CAS > 0 (The minimum number of CAS missions)

OCA < 96 (The maximum number of OCA missions)

AIRESC < 72 (The maximum number of AIRESC missions)

FSWP < 72 (The maximum number of FSWP missions)

BARCAP < 72 (The maximum number of BARCAP missions)

BAI < 98 (The maximum number of BAI missions)

INT < 96 (The maximum number of INT missions)

CAS < 216 (The maximum number of CAS missions)

The GINO problem was initally executed with the maximum number of aircraft within the theater equal to 56. Setting all the variables at the lowest level results in a total of 56 aircraft in the theater. The apportionment was recorded and then number of aircraft in the theater was increased by 20. Again, the problem was solved and the apportionment was recorded. Several starting values were used to avoid aircraft apportionment solutions that were only local optima. The process continued, adding 20 aircraft to the theater at a time, until the maximum number of aircraft in the theater was reached. The optimal apportionment allocated the next 216 aircraft to CAS missions. Once the number of aircraft in the theater exceeded 272 (216+56), available aircraft were assigned to INT missions. Finally, once the number of aircraft in the theater exceeded 368 (272+96), the AIRESC and FSWP missions were maximized. No other missions contributed to reducing the aircraft attrition.

Note that the same apportionment of 216 CAS, 96 INT, 72 AIRESC, 72 FSWP, 12 BARCAP, 8 SJAM, and 18 OCA aircraft optimizes all three MOEs! For this scenario, there was no tradeoff between air losses and ground losses or FLOT movement! The optimal apportionment begins with CAS, then INT, and finally a mixture of AIRESC and FSWP missions. CAS missions are the most effective mission for all three MOEs. INT missions are also effective in all three MOEs. The FSWP and AIRESC missions reduce the aircraft attrition while not having any significant effect on the other two MOEs. Although not specifically included in all the models, BARCAP, FSWP, AIRESC, SJAM, and OCA have positive minimum values. These terms would not appear as significant in the model if the minimum levels satisfied the requirements levied by the associated MOE's. SJAM and SSUP missions do not appear to have a significant impact on any of the MOE's examined in this study.

Figures 3.5, 3.6, and 3.7 show how the MOEs vary as the number of aircraft in the theater increases. These figures are based on the optimal allocation of aircraft given the number of aircraft in the theater. The solid squares on the charts indicate the average predicted response based on the number of aircraft in the theater. The hollow squares indicate a 95% confidence limit value. The 95% confidence limit values were calculated using Equation 3-5 with alpha equal to 0.1. Equation 3-5 is the equation for a two sided confidence limit of 90%. Since the normal function is symmetric, choosing the worst case boundary yields a 95% boundary. Since the experimental design was orthogonal, the covariance matrix is an identity matrix and Equation 3-5 reduces to 3-6.

$$\hat{Y} \pm t(n-p-1, 1-\frac{1}{2}\alpha) * s\sqrt{X_o' C X_o} \qquad \text{(Eq 3-5)}$$

Where:

X_O is a vector representing the apportionment of aircraft

$$\hat{Y} \pm t(n-p-1, 1-\frac{1}{2}\alpha) * s\sqrt{X_o' X_o} \qquad \text{(Eq 3-6)}$$

Figure 3.5 reveals that the metomodel predicts a negative number of aircraft lost when there are more than 400 aircraft in theater. Clearly, negative numbers of aircraft lost makes no sense. A reasonable interpretation of the chart is that the probability of losing more than 5 aircraft (out of 400+) is less than 5%. The model does not accurately estimate the mean number of aircraft lost at this extreme portion of the response surface.

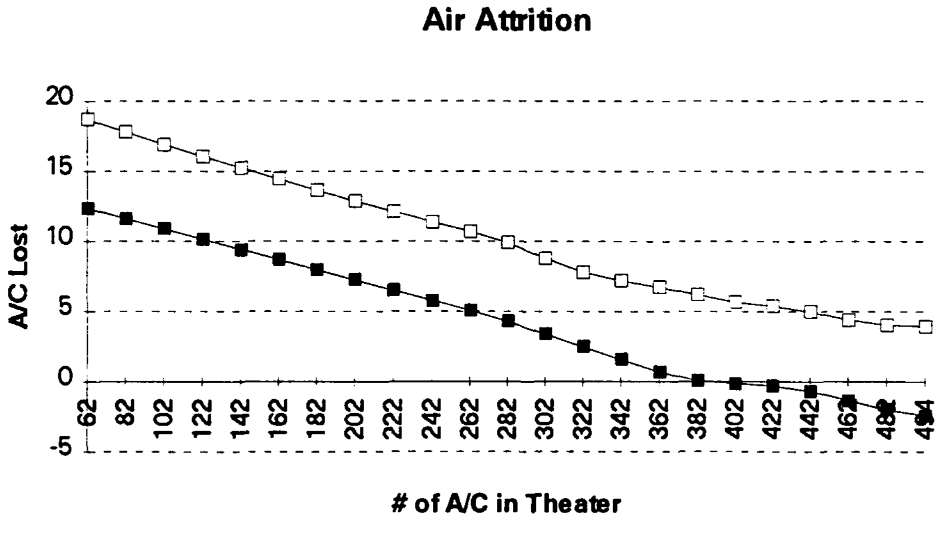


Figure 3.5 Aircraft attrition as a function of the number of aircraft in the theater based on the optimal apportionment of aircraft

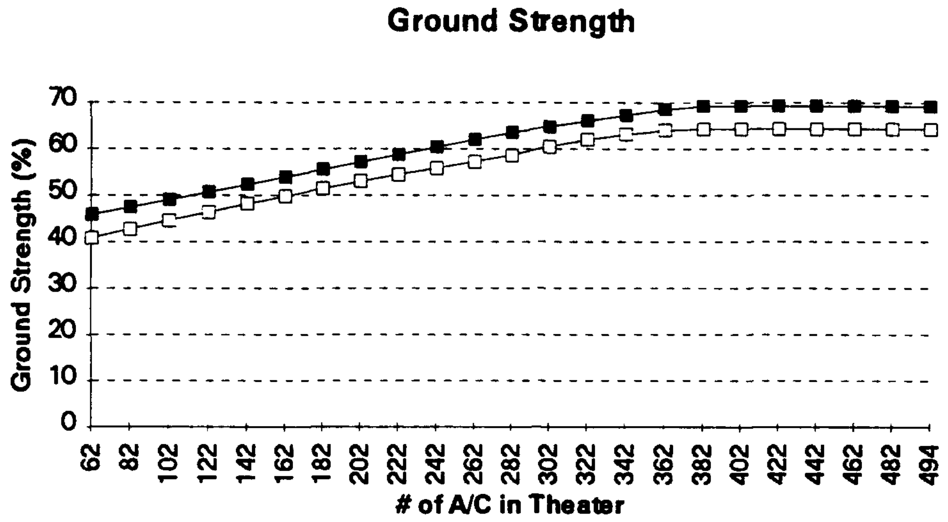


Figure 3.6 Remaining ground strength as a function of the number of aircraft in the theater

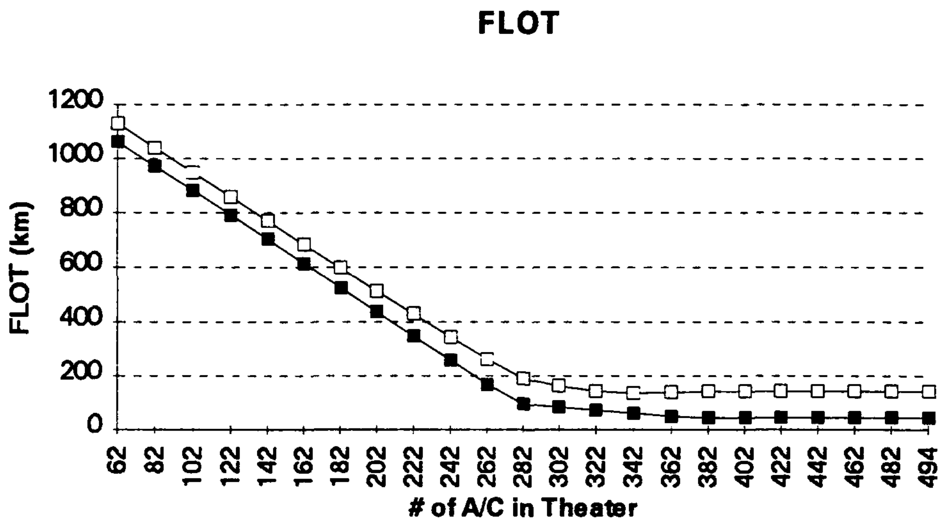


Figure 3.7 The total FLOT movement as a function of the number of aircraft in the theater

Figures 3.5 through 3.7 provide an analyst an upper bound on the MOEs as a function of aircraft in the theater assuming an optimal air apportionment. No non-material solution exists that will satisfy a requirement that exceeds the optimum MOE responses depicted in Figures 3.5 through 3.7.

Conclusion

It is important to emphasize that the results of this research cannot be generalized beyond the assumptions and limitations of the scenario examined. CAS missions seem to be very effective in reducing FLOT movement, reducing the attrition of front line troops, and reducing the number of aircraft lost. The first two results are expected since CAS missions attack opposing units on the front lines. The third result was unexpected. Is it due to the nature of this scenario or is it an anomaly of the TAC THUNDER model? Figures 3.3 through 3.5 show the predicted response based on the optimal allocation of aircraft. For this scenario, optimality is achieved by allocating aircraft first to CAS, then to INT, and finally to a even mix of FSWP and AIRESC. Exact values are calculated using GINO to optimize the allocation for a given number of aircraft assigned to the theater.

It should be noted that the minimum number of aircraft assigned to the OCA, BARCAP, FSWP, and AIRESC missions was set greater than zero. The absence of OCA and BARCAP from the optimal apportionment implies that assigning additional aircraft to these missions did not improve the MOEs examined in this study. It is unclear whether the minimum number of OCA and BARCAP aircraft was sufficient to perform a vital mission, or if these two missions did not contribute to accomplishing the overall objectives.

It is clear that RSM can estimate the response of TAC THUNDER to various air apportionments. Once the response surface has been estimated, the apportionment can be optimized and tradeoffs identified. The response surface also provides the analyst with insight into the performance of TAC THUNDER.

IV. Sensitivity Analysis

Introduction

The optimal solution found in chapter three is based on a fixed scenario. Many of the features of that scenario are not known exactly. A commander might well ask, "How sensitive is the solution to changes in this fixed scenario?" Sensitivity analysis can be used to investigate the effect of changes in the scenario on the "optimal" air apportionment.

The capabilities of men and machines must be quantified and input into combat simulations. Information on one's capabilities is usually more available than that of an adversary. Therefore, the estimates of an opponent's capabilities are less certain than the estimates of our own capabilities. The uncertainty about capabilities becomes even more pronounced when one is estimating the future capabilities of armed forces. The sensitivity analysis in this chapter focuses on two key parameters of the opponent's capabilities: their Air-to-Air Probability of Kill (AAPk) and their Surface-to-Air Probability of Kill (SAPk).

These values represent the probability that an enemy's attack will destroy an aircraft, given that it has been detected and attacked. Increasing these values represents an opponent with improved quality or utilization of its systems. In designing the experiment for this analysis, the high level for both the AAPk and SAPk values were established as twice their original values. This enhanced performance increased the enemy's Pk values to a level comparable with our own capabilities. The low level used was the same as in chapter three. Another candidate for sensitivity analysis was the set of Air-to-Surface Pk (ASPk) values. However, the ASPk values were already at parity with our own systems, and increasing them would not have been militarily realistic. Therefore, the Air-to-Surface Pk values are not part of the sensitivity analysis.

A second experiment was conducted, adding the AAPk and SAPk values to the seven significant factors found in the original response surface. New response surfaces were fit to estimate the relationship between the AAPk, SAPk, and an MOE. A worst case response surface was also developed for each MOE. The worst case response surfaces were optimized for each MOE using GINO.

These response surfaces indicate how TAC THUNDER responds to various inputs and provides useful information for understanding TAC THUNDER. Several non-intuitive effects were found which indicate the possibility that the TAC THUNDER algorithms may not be operating in a manner consistent with our understanding of combat.

Operational Uncertainty Analysis

A 2^{9-5}_{IV} design was used for this sensitivity analysis. Ten repetitions were again conducted at each of the 32 design points. A resolution four design is one in which the linear terms associated with each variable are confounded only with third and higher order terms. Some second order terms are confounded with other second order terms.

Appendix B describes how a simulated annealing program was developed to optimize the assignment of input variables to design factors A through I. A ten variable problem has approximately 10 factorial (approximately 4 million) combinations of possible assignments. This program attempts to find the assignment which minimizes the confounding of two factor interactions thought to be significant. Using this technique permited estimation of the selected two factor interactions with a resolution four experimental design in lieu of a more time consuming resolution 5 design. Appendix C contains the data used in this chapter.

Response surfaces were developed for each MOE using least squares regression. The model for the FLOT is:

$$FLOT = 979+(-2.05OCA+2.09FSWP+0.0435\mathrm{BARCAP})AAPk-2.52CAS-61.4SAPk-4.9INT+0.0426FSWP*BAI-.0192INT*OCA-0.050BAI*OCA \quad \text{(Eq 4-1)}$$

Where

AAPk is the level (-1 to +1) of the Air-to-Air Pk value.

SAPk is the level (-1 to +1) of the Surface-to-Air Pk value.

OCA is the number of OCA missions assigned to aircraft on the first day.

FSWP is the number of FSWP missions assigned to aircraft on the first day.

CAS is the number of CAS missions assigned to aircraft on the first day.

INT is the number of INT missions assigned to aircraft on the first day.

BAI is the number of BAI missions assigned to aircraft on the first day.

BARCAP is the number of BARCAP missions assigned to aircraft on the first day.

Negative one represents a Pk value equal to the baseline estimate. A positive one represents a Pk value equal to twice the baseline case. The ANOVA for the FLOT model in Equation 4-1 is given in Table 4.1.

Table 4.1 ANOVA table for FLOT (sensitivity model)

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
CAS	4137989	1	4137989	146	.0000
INT	695256	1	695256	26.3	.0000
SAPk	120786	1	120786	4.73	.0439
INT*OCA	124052	1	124052	4.69	.0414
BAI*OCA	136477	1	136477	5.16	.0332
AAPk*OCA	98014	1	98014	3.71	.0672
AAPk*FSWP	125325	1	125325	4.76	.0405
AAPk*BARCAP	125851	1	125851	4.76	.0405
FSWP*BAI	120565	1	120565	4.56	.0441
Total Error	536071	21	84135		
Total Corrected	659161	31			

$R^2 = 0.90$ R^2 (adjusted for degrees of freedom) = 0.87

Figure 4.1 shows how the blocking the observed FLOT movement by CAS and INT levels graphically depicts the impact these two variables have on the response. In Figure 4.1, a capital "C" or "I" indicates a high level of the CAS or INT variable. A lowercase "c" or "i" indicates a low level of the CAS or INT variable.

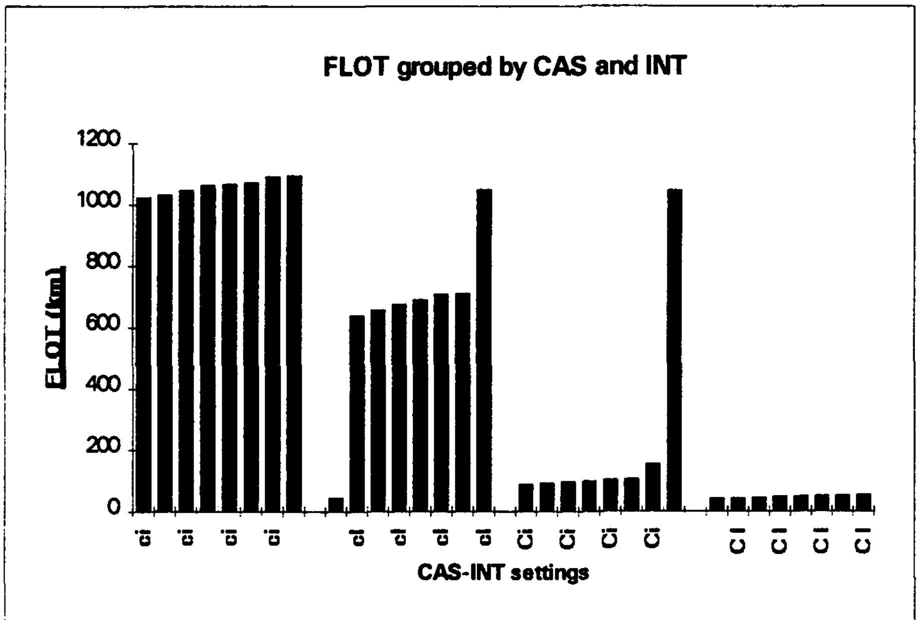


Figure 4.1 FLOT grouped by CAS and INT

Blocking the data can often yield better models Unfortunately, grouping the data this way does not really help to optimize the apportionment unless the optimization is restricted to a situation where the CAS or INT values are permanently set at their +1 or -1 values.

Figure 4.1 shows three uncharacteristic responses. If CAS or INT was the only variable at the high level, then a large FLOT response could still occur. Likewise, setting the other missions to their high level, except for CAS and INT, was sufficient to stop the Iraqis quickly. Figure 4.1 is useful in visually presenting the impact that CAS an[illegible] missions have on the FLOT MOE.

Similarly, the ground strength response surface was estimated as:

$$\begin{aligned} Grd\ Str = 85.53 &+ (0.0001757 AIRESC + 0.197 SAPk - 0.0078 BARCAP) AAPk - \\ & 0.0074 FSWP + 0.0064 CAS + 0.01956 BARCAP - 0.000489 BAI - \\ & 0.00511 OCA + 0.000248 BAI{*}OCA + \\ & SAPk(-0.0072 FSWP - 0.00154 CAS - 0.00306 INT) \qquad \text{(Eq 4-2)} \end{aligned}$$

Where:

Grd Str is the combat firepower of friendly ground units when phase one is completed.

BARCAP is the number of BARCAP missions assigned to aircraft on the first day.

AIRESC is the number of AIRESC missions assigned to aircraft on the first day.

Table 4.2 gives the ANOVA associated with the ground strength model associated with Equation 4-2.

Table 4.2 ANOVA for ground strength model (sensitivity analysis)

Input Variable	Sum of Squares	DF	Mean Sq.	F-Ratio	P-value
FSWP	1.5753	1	1.5753	11.3	.0035
CAS	15.2628	1	15.2628	109	.0000
BARCAP	3.5778	1	3.5778	25.7	.0001
INT	17.8503	1	17.8503	128	.0000
BAI	.812812	1	.8128	5.83	.0266
OCA	1.08781	1	1.0878	7.80	.0120
AAPk*AIRESC	2.0503	1	2.0503	14.7	.0012
AAPk*SAPk	1.24031	1	1.24031	8.89	.0080
AAPk*BARCAP	1.75781	1	1.75781	12.6	.0023
FSWP*SAPk	1.48471	1	1.48471	10.7	.0043
CAS*SAPk	.877813	1	.877813	6.29	.0239
SAPk*INT	.690313	1	.690313	4.95	.0391
BAI*OCA	.331553	1	.331553	23.8	.0001
Total Error	2.5106	18	.115943		
Total Corrected	54.0971875	31			

R^2= 0.9873 R^2 (adjusted for degrees of freedom) = 0.9781

Each of the terms in this model are significant at the 5% level. The R^2 and adjusted R^2 values are quite high. Figure 4.2 contains the normal plot of the residuals. The model appears to account for most of the significant structure in the data.

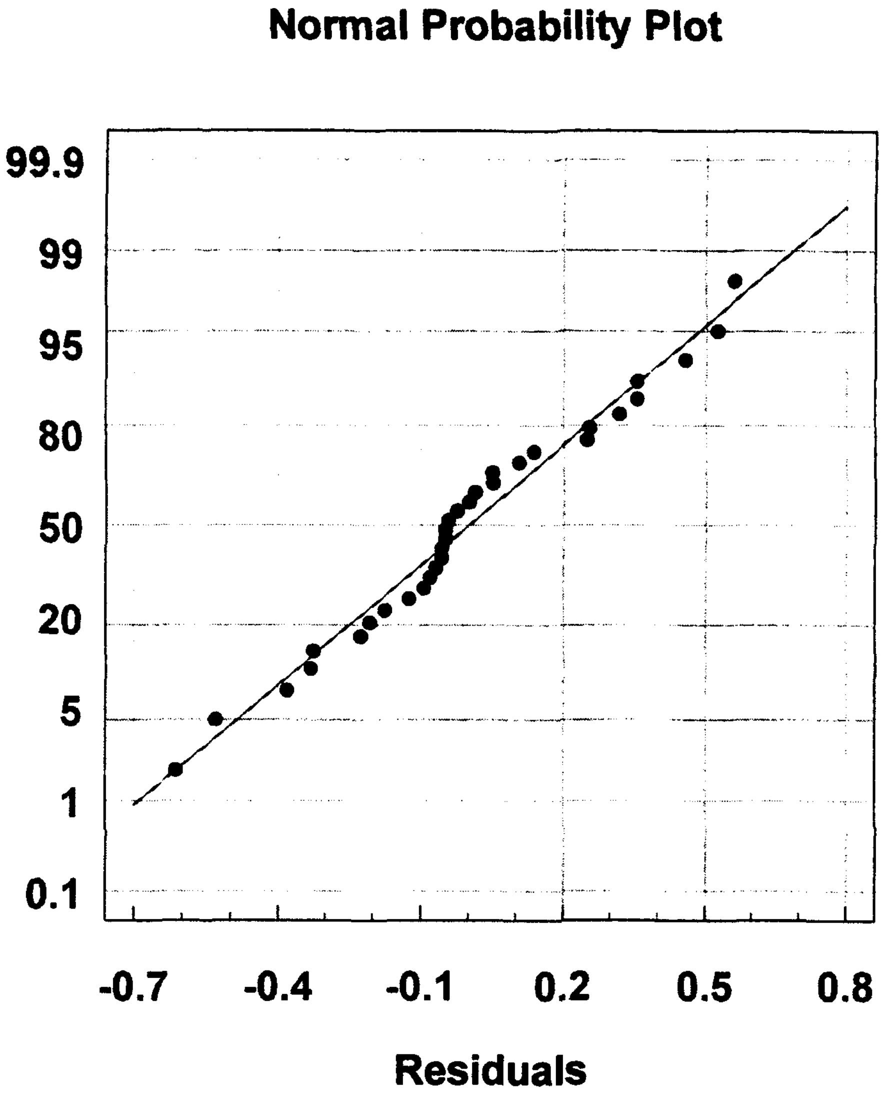


Figure 4.2 Normal plot of the residuals for the ground strength model (sensitivity analysis)

Figure 4.3 plots the residuals versus the expected value. This plot also indicates no significant structure can be observed to remain unaccounted for in the data.

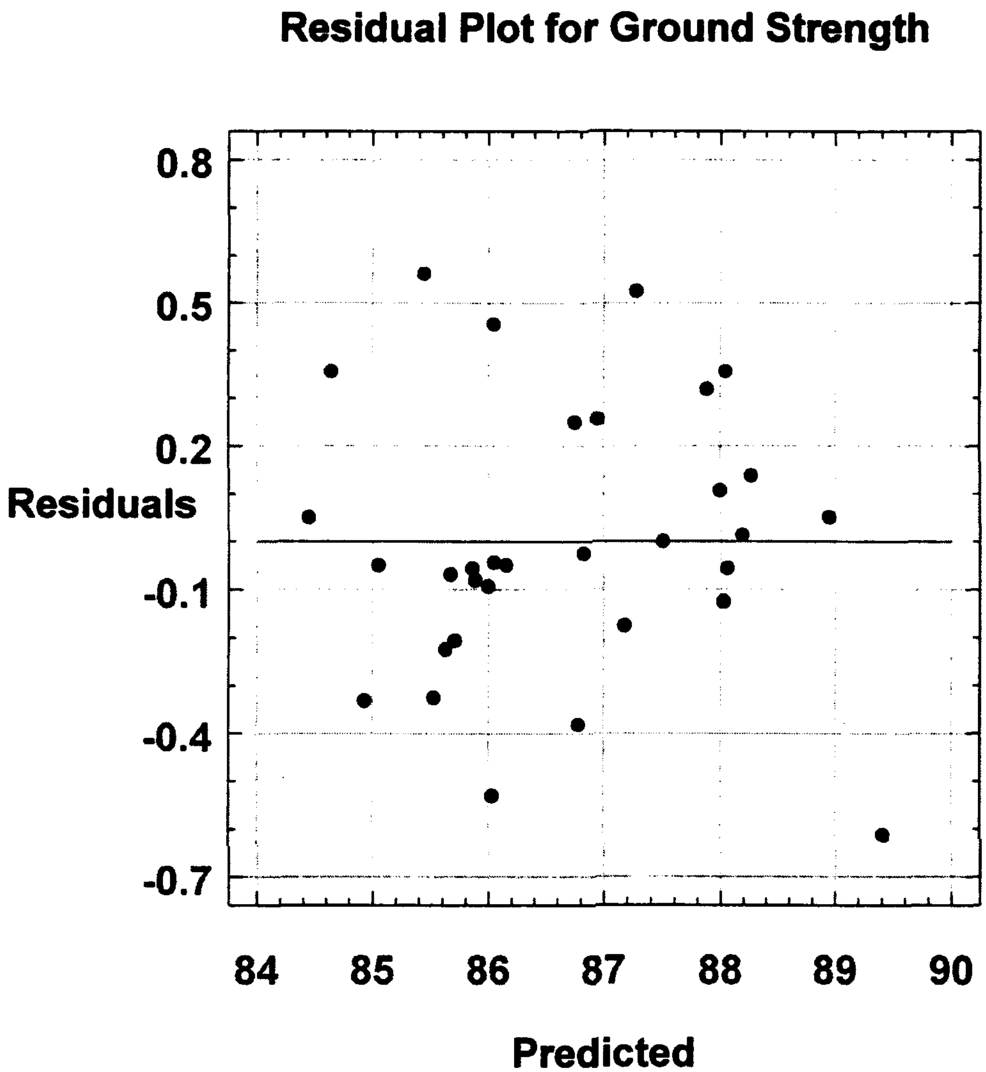


Figure 4.3 Plot of residuals vs. predicted values of the ground strength model (sensitivity analysis)

The air attrition response was the most difficult to estimate. The air attrition model was the most complex of the original experiment, and the two main effects added in the sensitivity experiment further complicated the model. Several alternatives were considered in developing a parsimonious representation of the simulation response. With respect to the air attrition MOE especially, the dividing line between significant and insignificant factors was not obvious. Judgment is a necessary element in this investigative process. The largest model considered included the mean and 30 other terms. The ANOVA for this model is presented in Table 4.3.

Table 4.3 ANOVA for air attrition

:

EFFECT	SUM OF SQUARES	DF	MEAN SQ	F-RATIO			
A:AAPk	129.605	1	129.605	3.096061	R-SQ:		0.990148
B:FSWP	310.005	1	310.005	7.405536			
C:AIRESC	10.58	1	10.58	0.25274	ADJ R-SQ:		0.694596
D:CAS	1576.411	1	1576.411	37.65801			
E:SAPk	195.0313	1	195.0313	4.658993	Mallow's CP:		31.10452
F:BARCAP	103.68	1	103.68	2.476754			
G:INT	41.405	1	41.405	0.989101	37.9 as s2		
H:BAI	3.645	1	3.645	0.087073			
I:OCA	197.0113	1	197.0113	4.706292			
AB+FG	11.52	1	11.52	0.275195			
AC+FH	79.38	1	79.38	1.896264			
AD+FI	1.71125	1	1.71125	0.040879	df of error	F of .1	F of .05
AE	0.45125	1	0.45125	0.01078	21	2.96	4.32
AF+BG+CH+DI	1.62	1	1.62	0.038699	20	2.97	4.35
AG+BF	73.205	1	73.205	1.748753	19	2.99	4.38
AH+CF	24.5	1	24.5	0.585267	16	3.05	4.49
AI+DF	90.45125	1	90.45125	2.160739	9	3.36	5.12
BC+GH	158.42	1	158.42	3.784407	5	4.06	6.61
BD+GI	270.2813	1	270.2813	6.456598			
BE	301.3513	1	301.3513	7.198812			
BH+CG	29.645	1	29.645	0.708173			
BI+DG	12.75125	1	12.75125	0.304607			
CD+HI	41.86125	1	41.86125	1			
CD	141.9613	1	141.9613	3.391233			
CI+DH	32.40125	1	32.40125	0.774015			
DE	52.02	1	52.02	1.242677			
EF	113.2513	1	113.2513	2.705396			
EG	0.15125	1	0.15125	0.003613			
EH	40.95125	1	40.95125	0.978262			
EI	162	1	162	3.869927			
TOTAL ERROR	41.86125	1	41.86125				
		31					

Note the summary measures in the upper right corner of the table. Mallow's Cp statistic requires an estimate of the variance of the process to be modeled. The variance used (37.9) is the average of the 10 replication variances computed for each of the 32 design points. Also note the critical F statistic values on the right side of the table. They provide the threshold criteria for determining the significance of individual terms in the model. Obviously some of these terms are not significant.

A useful means of comparing the candidate models is to plot the mean square error as the number of terms in the model increases. Figure 4.4 shows a plot of various air attrition models versus the mean square error of each model. Figure 4.4 helps to identify when enough terms have been added and additional terms only model the noise. It is possible to add terms that improve the fit for this particular data set which really have nothing to do with the process. Any model for air attrition involving more than 23 terms is certainly overfitting the data. Any model with less than 11 terms will have an MSE greater than 40, whereas the average sample variance for air attrition was 37.9.

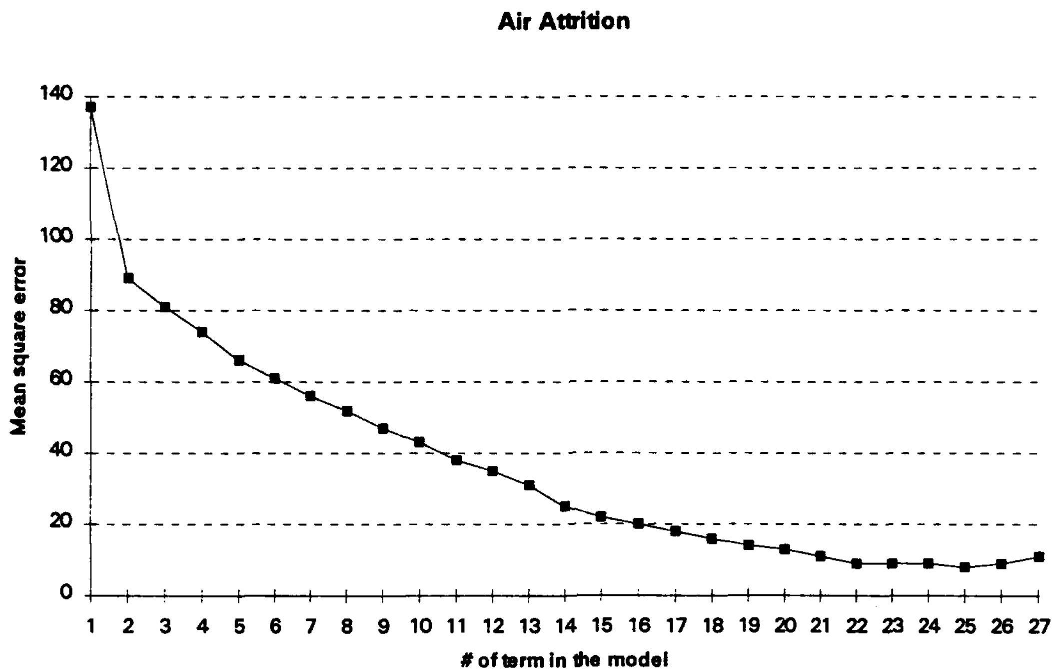


Figure 4.4 Graph of the MSE vs. # of terms in the model

Table 4.4 shows the ANOVA for the first candidate model for air attrition. It contains 23 terms including the constant term. The three candidate models were selected from those 11 to 23 terms. Model one had the highest R^2 value but some of the terms have F-ratios of less than 3.36. For this model, any term with an F-ratio of less than 3.36 fails the significance test at the 10% level.

Table 4.4 Model 1 for air attrition (sensitivity analysis)

ANOVA FOR AIR ATTRITION (SENSITIVITY STUDY)							
EFFECT	SUM OF SQUARES	DF	MEAN SQ	F-RATIO			
A:AAPk	129.605	1	129.605	13.83827	R-SQ:		0.980163
B:FSWP	310.005	1	310.005	33.10005			
C:AIRESC	10.58	0	0	0	ADJ R-SQ:		0.931671
D:CAS	1576.411	1	1576.411	168.3176			
E:SAPk	195.0313	1	195.0313	20.824	Mallow's CP:		16.22404
F:BARCA	103.68	1	103.68	11.07019	# of term:		23
G:INT	41.405	1	41.405	4.420922	37.9 as s2		
H:BAI	3.645	0	0	0			
I:OCA	197.0113	1	197.0113	21.03541			
AB +FG	11.52	0	0	0			
AC +FH	79.38	1	79.38	8.475613			
AD +FI	1.71125	0	0	0	df of error	F of .1	F of .05
AE	0.45125	0	0	0	21	2.96	4.32
AF +BG +CH +DI	1.62	0	0	0	20	2.97	4.35
AG +BF	73.205	1	73.205	7.816292	19	2.99	4.38
AH +CF	24.5	1	24.5	2.61593	16	3.05	4.49
AI +DF	90.45125	1	90.45125	9.65772	9	3.36	5.12
BC +GH	158.42	1	158.42	16.91492	5	4.06	6.61
BD +GI	270.2813	1	270.2813	28.85864			
BE	301.3513	1	301.3513	32.17607			
BH +CG	29.645	1	29.645	3.165275			
BI +DG	12.75125	0	0	0			
CD +HI	41.86125	1	41.86125	4.469637			
CD	141.9613	1	141.9613	15.15758			
CI +DH	32.40125	1	32.40125	3.459567			
DE	52.02	1	52.02	5.554313			
EF	113.2513	1	113.2513	12.09214			
EG	0.15125	0	0	0			
EH	40.95125	1	40.95125	4.372473			
EI	162	1	162	17.29717			
TOTAL ERROR	84.29125	9	9.365694				
		31					

Table 4.5 is the ANOVA table for the second candidate model for air attrition. Since it has fewer terms than the previous model, its R^2 value is lower. Each of the terms in this model passes the F-test for significance at the 10% level. But not at the 5% level.

Table 4.5 Model 2 for the air attrition (sensitivity analysis)

ANOVA FOR AIR ATTRITION (SENSITIVITY STUDY)										
EFFECT		SUM OF SQUARES		DF	MEAN SQ	F-RATIO				
A:AAPk		129.605		1	129.605	5.974732		R-SQ:		0.918318
B:FSWP		310.005		1	310.005	14.29109				
C:AIRESC		10.58		0	0	0		ADJ R-SQ:		0.841742
D:CAS		1576.411		1	1576.411	72.67184				
E:SAPk		195.0313		1	195.0313	8.990852		Mallow's CP:		9.157652
F:BARCAP		103.68		1	103.68	4.779601		# of term:		16
G:INT		41.405		0	0	0		37.9 as s2		
H:BAI		3.645		0	0	0				
I:OCA		197.0113		1	197.0113	9.082129				
AB+FG		11.52		0	0	0				
AC+FH		79.38		1	79.38	3.659382				
AD+FI		1.71125		0	0	0		df of error	F of .1	F of .05
AE		0.45125		0	0	0		21	2.96	4.32
AF+BG+CH+DI		1.62		0	0	0		20	2.97	4.35
AG+BF		73.205		1	73.205	3.374717		19	2.99	4.38
AH+CF		24.5		0	0	0		16	3.05	4.49
AI+DF		90.45125		1	90.45125	4.169762		9	3.36	5.12
BC+GH		158.42		1	158.42	7.30309				
BD+GI		270.2813		1	270.2813	12.45984				
BE		301.3513		1	301.3513	13.89216				
BH+CG		29.645		0	0	0				
BI+DG		12.75125		0	0	0				
CD+HI		41.86125		0	0	0				
CD		141.9613		1	141.9613	6.544349				
CI+DH		32.40125		0	0	0				
DE		52.02		0	0	0				
EF		113.2513		1	113.2513	5.220831				
EG		0.15125		0	0	0				
EH		40.95125		0	0	0				
EI		162		1	162	7.468126				
TOTAL ERROR		347.075		16	21.69219					
				31						

Table 4.6 is the ANOVA for the third candidate air attrition model and has the fewest number of terms considered. All the terms are also significant at the 10% level. This model has two advantages over the two previous models. It is simpler and the estimate for the variance of the air attrition process matches the average variance of the TAC THUNDER runs.

Table 4.6 Model 3 for air attrition (sensitivity analysis)

ANOVA FOR AIR ATTRITION (SENSITIVITY STUDY)							
EFFECT	SUM OF SQUARES	DF	MEAN SQ	F-RATIO			
A:AAPk	129.605	1	129.605	3.372443	R-SQ:		0.810068
B:FSWP	310.005	1	310.005	8.06662			
C:AIRESC	10.58	0	0	0	ADJ R-SQ:		0.719625
D:CAS	1576.411	1	1576.411	41.01969			
E:SAPk	195.0313	1	195.0313	5.074895	Mallows CP:		11.294
F:BARCAP	103.68	0	0	0	# of term:		11
G:INT	41.405	0	0	0	37.9 as s2		
H:BAI	3.645	0	0	0			
I:OCA	197.0113	1	197.0113	5.126417			
AB+FG	11.52	0	0	0			
AC+FH	79.38	0	0	0			
AD+FI	1.71125	0	0	0	df of error	F of .1	F of .05
AE	0.45125	0	0	0	21	2.96	4.32
AF+BG+CH+DI	1.62	0	0	0	20	2.97	4.35
AG+BF	73.205	0	0	0	19	2.99	4.38
AH+CF	24.5	0	0	0	16	3.05	4.49
AI+DF	90.45125	0	0	0	9	3.36	5.12
BC+GH	158.42	1	158.42	4.122236			
BD+GI	270.2813	1	270.2813	7.032971			
BE	301.3513	1	301.3513	7.841441			
BH+CG	29.645	0	0	0			
BI+DG	12.75125	0	0	0			
CD+HI	41.86125	0	0	0			
CD	141.9613	1	141.9613	3.693964			
CI+DH	32.40125	0	0	0			
DE	52.02	0	0	0			
EF	113.2513	0	0	0			
EG	0.15125	0	0	0			
EH	40.95125	0	0	0			
EI	162	1	162	4.215391			
TOTAL ERROR	807.0425	21	38.4306				
		31					

The third model has a mean square error of 38.4. This value agrees nicely with 37.9, which is the estimate from the 10 repetitions at each of the 32 design points. Model three was chosen because the MSE is approximately equal to the variance of the data. All of its terms are significant at the 10% level.

The ANOVA tables show which two factor interactions are aliased with other two factor interactions. Only one of each confounded pair is believed to be potentially significant. Interactions included in the first design and shown to be insignificant were assumed to continue to be insignificant. By allowing this two factor aliasing, the significant two factor effects could be estimated without increasing the resolution of the experiment to a resolution five design. A resolution five design would have required twice as many design points but no two factor interactions would have been aliased with any other two factor interaction.

The equation resulting from the third model is:

$$Air\ Atr = 10.95 +(-1.0316+.1023FSWP-.0833OCA+.0703AIRESC)SAPk+2.013*AAPk+ .2075FSWP-.006499CAS+.1009INT+.1995OCA-.002242INT*OCA- .00247FSWP*AIRESC+.1038AIRESC \quad \text{(Eq 4-3)}$$

Figure 4.5 shows a plot of the residuals of this air attrition model plotted against the predicted value. No significant pattern was observed in Figure 4.5 The presence of such a pattern would be evidence in favor of a model with more terms.

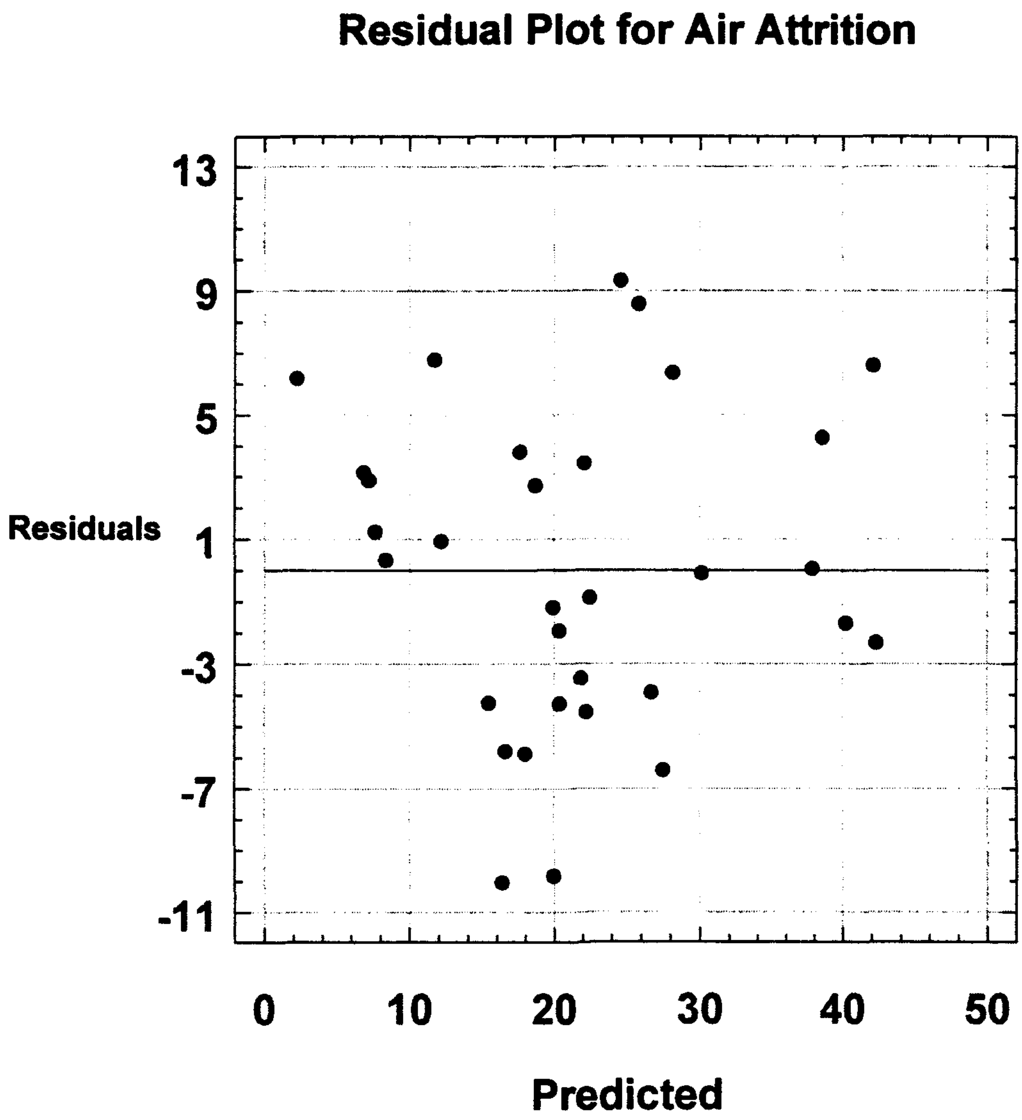


Figure 4.5 Plot of the residuals vs. the predicted values for the Air Attrition model (sensitivity analysis)

Figure 4.6 shows a normal probability plot of the residuals of the air attrition model. The presence of data points which do not lie along the line indicates that some additional structure remains unexplained by the current model.

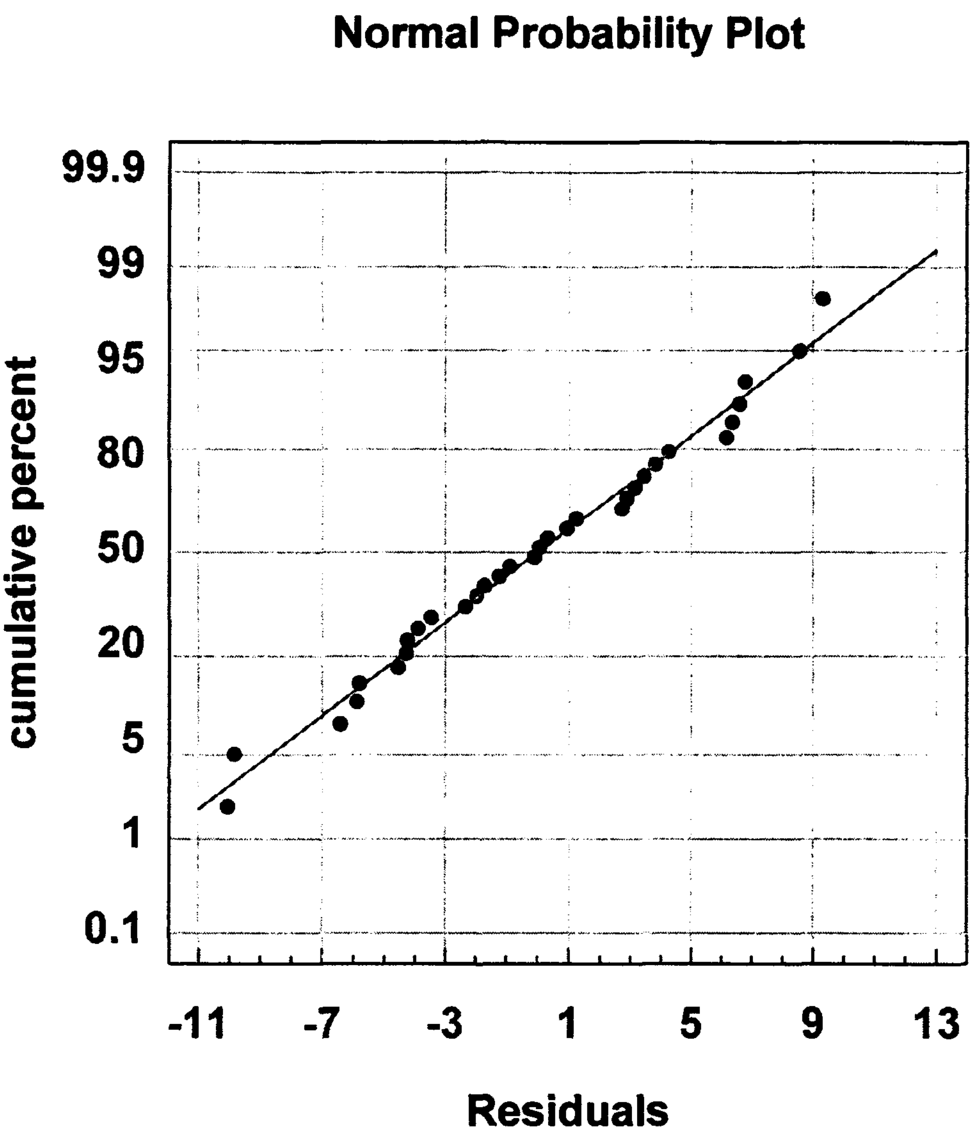


Figure 4.6 Normal plot of the residuals of the Air Attrition model (sensitivity analysis)

The model selected contains the most significant terms and predicts a variance approximately equal to the estimate of the variance found in the data. However, the difficulties in selecting a "good" air attrition model indicate that the air attrition process is complex.

Analysis

The three models developed from the sensitivity experiment form the basis for the analysis of the relationship between the various outputs and the two operational uncertainty variables. The partial derivative of each model with respect to an operational uncertainty variable indicates how that MOE tends to change as the uncertainty variable increases. The partial derivatives represent the response of the MOE to changes in the uncertainty variables; an important aspect in analyzing the behavior of TAC THUNDER. Intuitively, it is expected that as the opponents become more capable, their results should improve.

The partial derivative of the FLOT with respect to AAPk is :

$$\frac{\partial FLOT}{\partial AAPk} = -2.05OCA + 2.09FSWP + .0435BARCAP \qquad \text{(Eq 4-4)}$$

OCA missions become more effective as the opponents aircraft become more lethal in air-to air engagements. Historically, it is easier to kill aircraft on the ground than in air-to-air combat. FSWP and to a lesser extent BARCAP missions become less useful in the sense that they generate more losses per mission than previously. It is interesting to note that there is not a significant decrease in the effectiveness of CAS as the AAPk increases.

The partial derivative of the FLOT with respect to SAPk is :

$$\frac{\partial FLOT}{\partial SAPk} = -61.4 \qquad \text{(Eq 4-5)}$$

Figure 4.7 shows the FLOT observations grouped by the low and high values of the SAPk and arranged in ascending order. As seen in Figure 4.7, the net effect of doubling the opponent's SAPk was that they made less progress on the ground! This response is very nonintuitive. One possible explanation lies in the sortie assignment algorithm of TAC THUNDER. The increased SAPk value might change the way TAC THUNDER organized the strike packages which attacked ground targets.

If the increased SAPk value (which the algorithm uses!) changed the way the aircraft were assigned into packages and these packages assigned to targets, then perhaps these new assignments were more effective at stopping FLOT movement. Clearly something unusual is going on here.

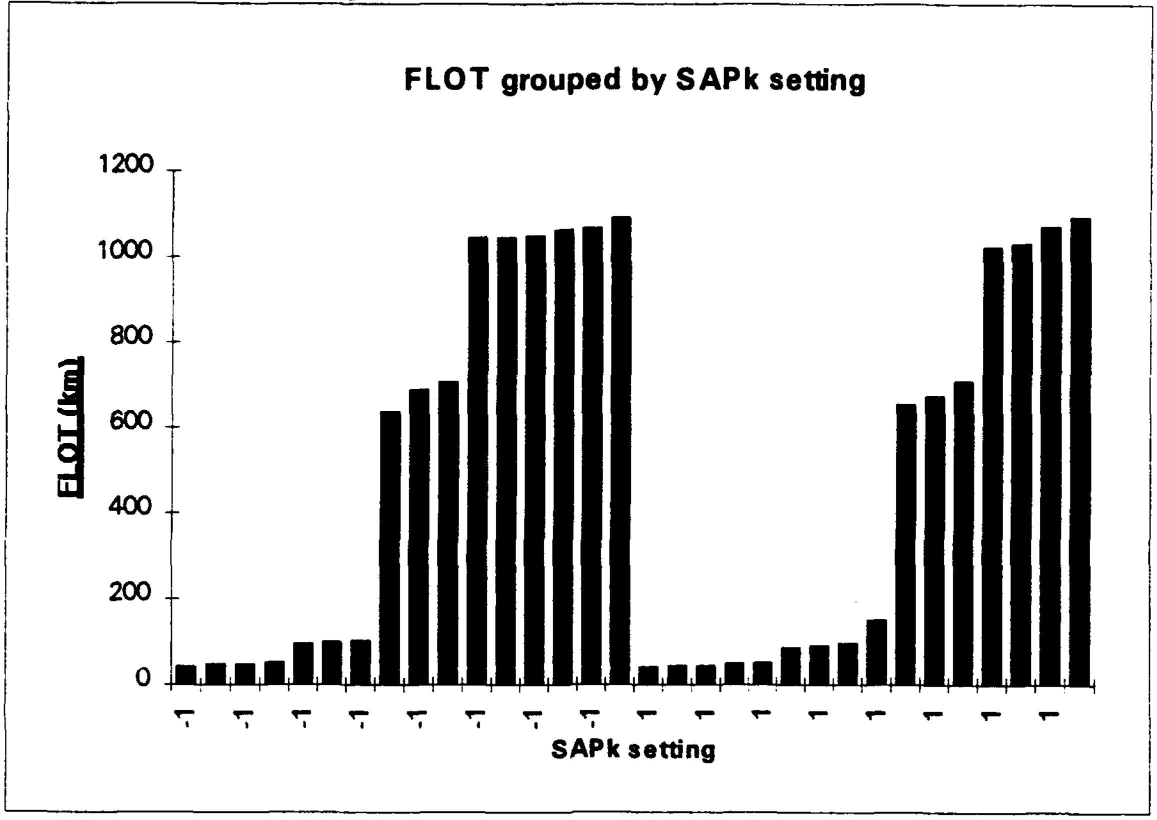


Figure 4.7 FLOT vs. SAPk values

The partial derivative of the ground strength with respect to AAPk is:

$$\frac{\partial GRDSTR}{\partial AAPk} = -0.000176 AIRESC + .197 SAPk - 0.0078 BARCAP \qquad \text{(Eq 4-6)}$$

AIRESC and BARCAP both become less effective (for maximizing the ground strength) when the AAPk values increase.

The partial derivative of the ground strength with respect to SAPk is:

$$\frac{\partial GRDSTR}{\partial SAPk} = -0.0072 FSWP - 0.00154 CAS - 0.00306 INT + 0.197 AAPk \qquad \text{(Eq 4-7)}$$

As the opponent's SAPk value increases, the contribution made by each FSWP, CAS, and INT sortie is reduced.

The partial derivative of the air attrition with respect to SAPk is:

$$\frac{\partial AIRATR}{\partial AAPk} = 2.0125 \qquad \text{(Eq 4-8)}$$

When the opponent's AAPk value changed from the low setting to the high setting, an average of four additional aircraft were lost.

The partial derivative of the air attrition with respect to SAPk is:

$$\frac{\partial AIRATR}{\partial SAPk} = -1.0316 + 0.102FSWP - 0.0833OCA + .07033AIRESC \qquad \text{(Eq 4-9)}$$

As the SAPk value increases, the value of OCA missions decreases. This response is reasonable since airfields tend to be heavily defended against aircraft attacks. In contrast, the value of FSWP and AIRESC missions increases as the enemy's SAPk increases. It may be that these aircraft are more survivable against surface-to-air threats. If attacks are made against these sorties instead of other, more vulnerable, sorties, then the average number of aircraft shot down would be reduced.

Worst Case Analysis

This section considers how the optimal apportionment found in chapter three performs against a worst case combination of the SAPk and AAPk values. Simply setting SAPk and AAPk to their highest level might not be the worst case. Worst case versions of Equations 4-1 through 4-3 were found by selecting the worst value for each term in the model. For example, the coefficient associated with OCA in Equation 4-3 is 0.1995 +/- 0.0833, depending on the value of the SAPk term. For air attrition, more aircraft are lost as a coefficient increases. Therefore, 0.2828 was the value used for the OCA coefficient in the worst case air attrition model. Using this technique, no matter what the apportionment, these new equations will represent the worst possible values.

FLOT Worst case:

$$FLOT= 1045+.0435BCAP-2.52CAS-4.9INT+.03FSWP+.0426FSWP*BAI-.0192INT*OCA-.050BAI*OCA+4.47OCA-.4765BAI \quad \text{(Eq 4-10)}$$

Ground Strength Worst case:

$$Grd\ Str = 85.38 -.0001757AIRESC-.0144FSWP+.0049CAS+.001176BARCAP-.000489BAI-.00511OCA+.000248BAI*OCA+ .306INT) \quad \text{(Eq 4-11)}$$

Air Attrition Worst case:

$$Air\ Atr = 14 +.3098FSWP-.006499CAS+.1009INT+.2828OCA-.002242INT*OCA-.00247FSWP*AIRESC+.174AIRESC \quad \text{(Eq 4-12)}$$

Optimizing these equations for their respective MOEs yields the best worst case apportionment for each MOE. Figures 4.8 to 4.10 present the results of these apportionments. Unlike the apportionment in chapter three, each of the three MOEs has a different optimum apportionment. Figure 4.8 shows the predicted FLOT as a function of the number of aircraft in the theater. The aircraft have been optimized for each of the three MOEs. For a given number of aircraft, the top bar represents the optimal apportionment to minimize the loss of aircraft. The middle bar represents the optimal apportionment to maximize the ground strength. The bottom bar represents the optimal apportionment to minimize the FLOT movement.

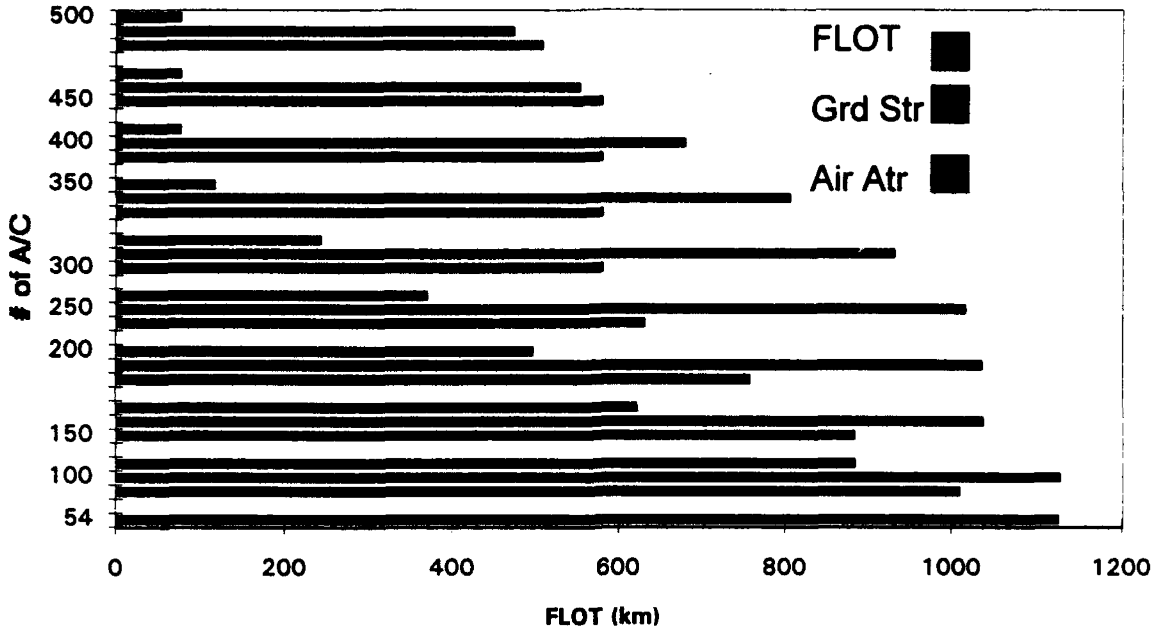


Figure 4.8 FLOT outcomes as a function of the apportionment optimized for air attrition, ground strength, and FLOT

Similarly Figure 4.9 shows the predicted ground strength value of the three sub optimized air apportionments.

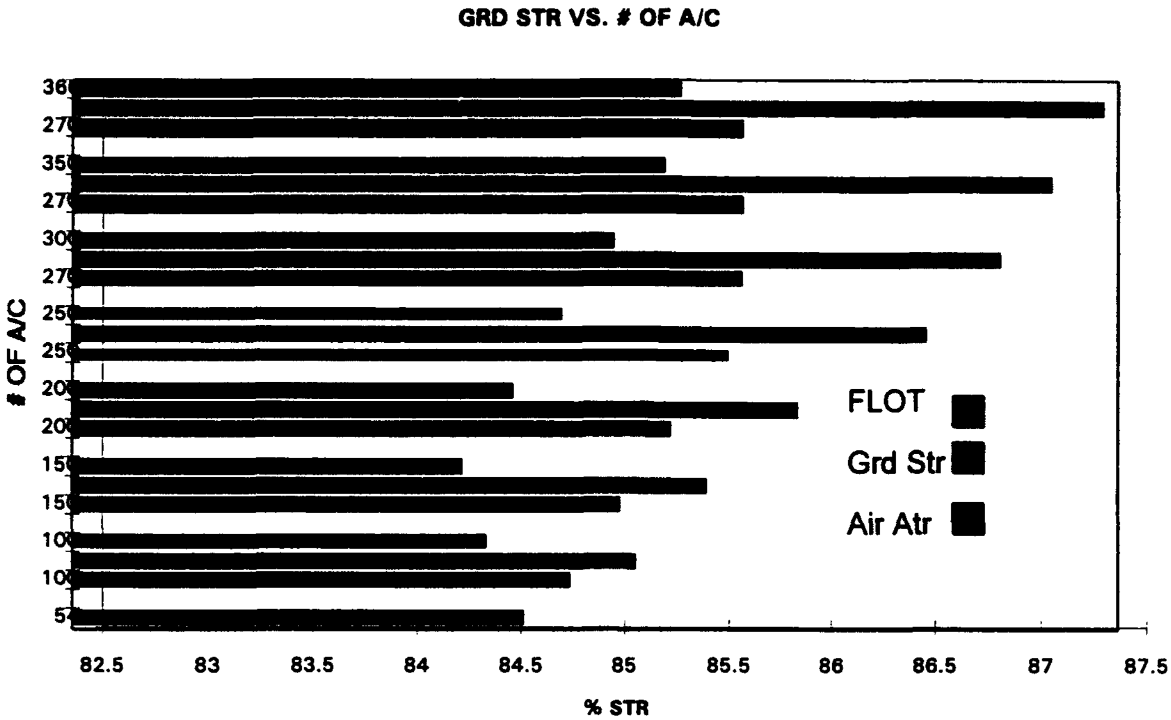


Figure 4.9 Ground strength comparisons based on # of A/c Figure 4.10 shows the predicted aircraft attrition as a function of the number of aircraft available.

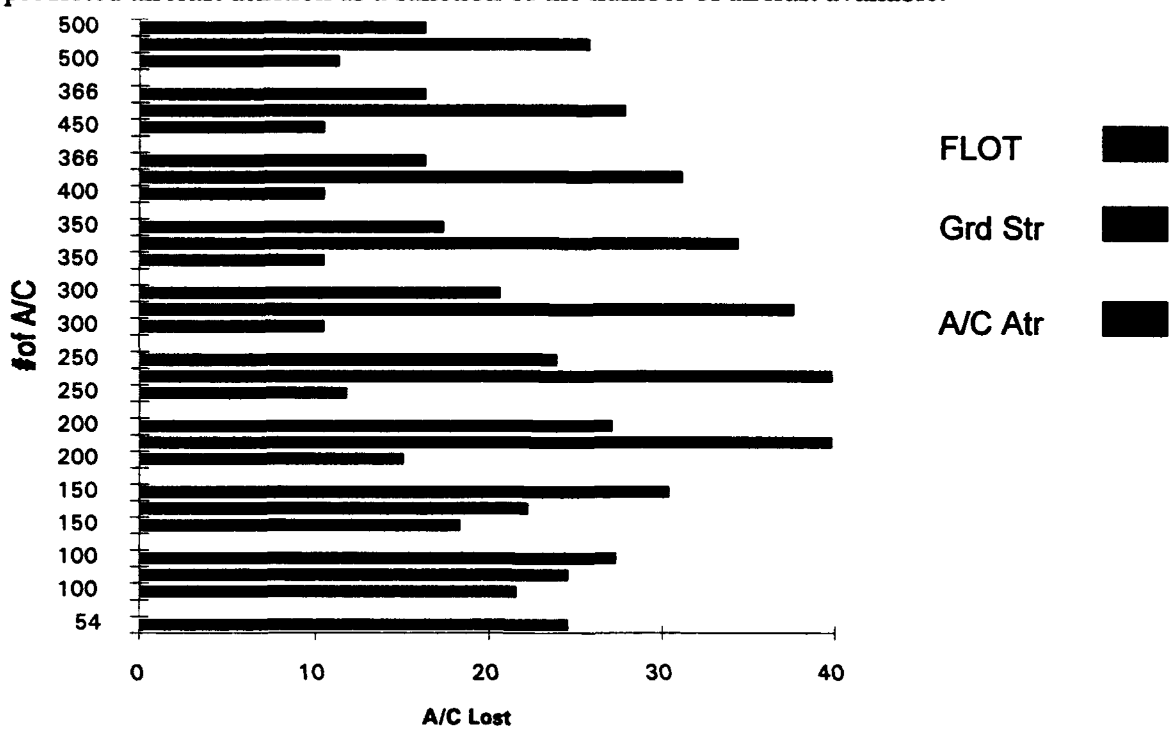


Figure 4.10 Aircraft attrition as a function of # of aircraft and MOE optimized

Figures 4.8 to 4.10 show how optimizing the air attrition or ground strength causes a large FLOT result. For this scenario at least, a low casualty rates should not be the primary objective. Minimizing the FLOT movement does not result in a large increase in air or ground losses.

The sensitivity models were compared to the baseline models by substituting -1 for the SAPk and AAPk terms in Equations 4-1, 4-2, and 4-3. The predicted values for the three MOE's were generated for 26 design points. These design points were the 16 used in chapter three and the eight design points in chapter four that had SAPk and AAPk settings of -1. Ideally, the predictions of the baseline models would be highly correlated with the predictions of the sensitivity models. The correlations between the baseline models and the sensitivity models for the air attrition, ground strength, and FLOT MOE's were .52, .61, and .76, respectively. Appendix D contains plots of the baseline models' predictions versus the sensitivity models' predictions. No nonlinear relationship was observed in these plots. Several interesting features were observed but the timing of this research precluded further investigation. Apparently, there were significant differences in TAC THUNDER's responses that were not accounted for in the SAPk and AAPk terms.

Conclusion:

CAS missions proved beneficial under the range of operational uncertainty variables. No other mission category was uniformly beneficial under an absolute worst case scenario. Interdiction could also help the FLOT MOE while only increasing the air attrition slightly. In view of these results, the baseline apportionment found in chapter three is still valid and fairly robust. After these two missions, the baseline added a mix of air escort and interdiction missions. Under the worst case analysis, these missions reduced the overall effectiveness of the apportionment.

Response surface models allow a quick response to what-if questions without re-running a time-consuming model. As long as the region of interest has been covered in the design, RSM gives the analyst insight into the behavior of the model and a better ability to respond to new questions by the decision maker.

Such insight into TAC THUNDER's behavior would be quite useful in the verification, validation, and accreditation (VV&A) process. A user (or designer) of a model wants to know that the model behaves in a reasonable, rational way. It must simulate the process in a manner appropriate for the intended application.

Several interesting observations were made by analyzing how TAC THUNDER responds to increases in the probability of kill values. When the Pk values for the surface to air threats were increased for the opponents, the opponents had less success with the FLOT MOE. Intuitively one would expect the opponents to make more progress. One possible explanation is that the sortie generation algorithm used the change in Pk values to change the way the sorties were organized and flown. The goal of the sensitivity experiment was to observe the effect on the baseline case, given that the opposing force was more effective than anticipated. If the algorithm behaves in such a manner, it is impossible to change how well the opponents fight without changing our perception of their capabilities. Another possibility is that there could be a problem with the algorithms that use the SAPk values. One experiment does not prove anything, but it does identify an area of concern. Additional work should be done to understand how TAC THUNDER behaves. It is possible that our intuition is wrong and these results might give us better insight into theater level combat. That is one of the primary purposes of all models: to act as a catalyst for insight.

V. Conclusions and Recommendations

Summary

The purpose of this thesis was to demonstrate that RSM techniques could be used to optimize the air apportionment in a TAC THUNDER scenario. An unclassified scenario was developed and its air apportionment optimized for three MOEs: FLOT, ground strength, and air attrition. The screening design provided sufficient data to model the relationship between the air apportionment and the three MOEs. The metamodels which quantified the relationship between the model inputs and the MOEs were optimized with constraints appropriate to the study. These metamodels allow an analyst to quickly answer what-if questions from the decision maker as long as the questions concern the region bounded by the original design space.

In the first phase of this investigation (chapter three), one apportionment optimized all three MOE's. The baseline apportionment, in rank order, is as follows:

1 - Close Air Support

2 - Interdiction

3 - An even mix of Air Escort and Fighter Sweep missions.

The Air Escort and Fighter Sweep missions reduced the air attrition while having no significant effect on the other two MOEs.

The second phase in this investigation examined the sensitivity of the response surfaces to two operational uncertainty variables. Close Air Support proved to be useful in all cases for all three MOEs. The response of the three MOEs to changes in the Surface-to-Air Probability of Kill (SAPk) value was quite interesting. As the opponents SAPk increased, they made less progress on the ground even though they did shoot down more of our aircraft. An explanation for this result needs to be found. This is an excellent example of how RSM can provide understanding of the behavior of complex simulations thereby supporting the verification, validation, and accreditation (VV&A) process.

Recommendations for Follow-On Efforts

It is difficult (and incorrect) to generalize with data from only one phase of one scenario. Additional study needs to be done on other phases of this scenario. Similar studies using different scenarios would help to generalize the results and support efforts to use TAC THUNDER to examine AF doctrine. The sensitivity of the air apportionment to changes in the opponent's air apportionment needs to be studied. The new Theater-Level Model, being developed at the Naval Post-Graduate School, uses a simpler air apportionment that would be ideal for studying the air apportionments of opposing forces using RSM and game theory. Classified studies could be done to see if the results corroborate this study. A detailed study could be made of how well the metamodels predicted the TAC THUNDER output.

An important area of study is multi-phase operations. Is it appropriate to optimize each phase of a campaign separately? Or, does the entire campaign need to be optimized simultaneously? How can a multi-phase campaign be optimized with limited time and computing resources?

Finally, VV&A efforts on combat models could incorporate RSM techniques. It is difficult, if not impossible, to prove that a model "correctly" models theater-level warfare. Aggregation and assumptions are fundamental components of a theater-level combat simulation. Identifying the relationship between the inputs and outputs of a complex simulation should provide evidence that the model is working as expected or that the models behaves in unexpected ways. In that case, either the model or our expectations would need to be changed.

Stochastic annealing proved an effective way to match the variables to the factors in the experimental design stage. This technique may allow experiments with large numbers of variables to reduce the number of design points needed; if some of the factors can be considered negligible, then a design that does not alias any of the nonnegligable terms may be sufficient. Additional work needs to be done in this area.

Conclusions

This thesis has demonstrated that RSM techniques can be used to optimize the air apportionment in a TAC THUNDER scenario. It is limited to those cases where the number of variables can be reduced to a manageable level, such as was done here by grouping the aircraft and optimizing only one phase of a campaign. Since the best use of force is required to study tradeoffs between force mixes, this technique could support many combat effectiveness studies and acquisition decisions.

APPENDIX A

This appendix provides a description of the TAC THUNDER scenario used in this thesis effort. It will describe the order of battle for both sides including units, squadrons, number and type of aircraft.

Those files modified from the standard unclassified Middle East scenario are also provided.

The order of battle was:

UNIT	TANKS	APC	HELO	ARTY	INF	AIR DEF
1St Saudi Div	348	318	42	132	1000	12
82nd Airborne	0	270	42	54	1500	12
24th Mech Inf Div	290	270	42	126	1000	12
1St MEF	348	318	42	132	1000	12
TOTAL	986	1176	168	444	4500	48

AIRCRAFT:

F-15	36-216
F-15E	0-194
A-10	0-216
EF111	8-30
F111	18-72
WEASEL	0-24

IRAQI FORCES:

UNIT	TANKS	APC	HELO	ARTY	INF	AD GUN	SAM
ARMOR DIV (x7)	325	250	12	150	1000	16	8
MECH DIV (x3)	225	650	12	150	1200	16	8
INF DIV (x8)	90	150	0	150	3000	16	0
TOTAL	3670	4900	120	2700	34600	288	80

AIRCRAFT:

MIG29	25
MIG23	100
MIG21	150
MIRAGE F1	125
SSU25	50

AIR DEFENSE COMPLEXES: 10 SA-3 SITES

The sortie rates used included a maximum surge rate for the first six days of the conflict and a maximum sustained rate. These sortie rates by aircraft were:

A/C	DAY.IN.THEATER.	.AUTH.QTY.SORT/DAY	..AC.MAX.SORT/DAY
A-10	1.00	3.00	4.00
	6.00	2.00	3.00
RF-4	1.00	2.50	3.00
	6.00	1.50	2.00
F-111	1.00	2.00	2.50
	6.00	1.20	1.50
F-15	1.00	2.50	3.00
	6.00	2.20	2.50
MIG-23	1.00	3.00	3.00
	6.00	1.20	1.20
MIRAGE F-1	1.00	3.50	4.00
	6.00	2.70	2.70
MIG-21	1.00	3.00	3.00
	6.00	1.20	1.20
MIG-29	1.00	3.00	4.00
	6.00	2.50	2.70
SU-25	1.00	2.20	2.20
	6.00	.80	.80

APPENDIX B

This appendix provides a description of the simulated annealing code developed in this thesis effort. The code is also provided.

BACKGROUND

High resolution designs for experiments with a few variables are simple to design and carry out. As the number of variables increase, the number of individual experiments needed to achieve the same level of resolution increases rapidly. With 10 variables, a fractional factorial design requires 64 runs to achieve resolution five. If some of the two variable interactions are known (or assumed) to be insignificant, then a smaller design can be used where none of the significant two variable interactions are confounded with any of the other two variable interactions. This allows a reduction in the resources required to carry out the experiment.

Each variable in the experiment must be assigned to one of the columns of the experimental design matrix. For a given design, the confounding of each factor is fixed. For example, Factor A*Factor B is confounded with Factor I*Factor J. If the variables are assigned to factors A, B, I, and J such that AB and IJ are not both significant, than this confounding is not a problem. The number of ways 10 variables can be assigned to 10 factors is 10! or approximately 4 million combinations.

Simulated Annealing

Simulated annealing is a recent technique inspired by the physical process in metals. This technique uses a temperature function that is analogous to the temperature in the cooling metal. As the temperature drops, the atoms in the metal tend to arrange themselves in lower energy states. In this implementation, the energy state is the value of the objective function.

Each variable pair represents an important effect. Weights are assigned to each pair that correspond to the magnitude of their significance. These weights are assigned by the experimenter based on his knowledge of the process under study and previous experiments. Each assignment of variables to factors has a value equal to the sum of the weights of all the confounded pairs. If none of the important variable pairs are confounded with other variable pairs, then the value of that assignment is zero. A value of zero is the best possible value. It may not be possible to deconflict every variable pair. The researcher could accept a sufficiently low value if appropriate.

The Code

The following is a description of each section of the code:

1-90	Define the variables.
100-190	Define the initial assignment
200-405	Define the important variable pairs.
410- 500	Define the weights associated with each important variable pair.
1000-3390	Check each set two factor interactions. If more than one is important then add the sum of their weights to the objective function.
3400-3490	Implement the temperature function that controls how many iterations will be made at one temperature setting. Once the correct number of iterations have been made, the temperature decreases.
4000-4005	If the test assignment has a lower value than best assignment found so far, it becomes the new best assignment.
4010	If the test assignment is accepted as the new current assignment, then it becomes the new current assignment.
5000-6000	Generate a new test assignment by perturbing the current assignment. First, variable assignments are switched. Two variables are selected randomly and switched repeatedly.

	The number of exchanges are a function of the temperature. At high temperatures the mean number of switches is large. As the temperature falls, the average perturbation becomes smaller.
10000-10200	Subroutine which updates the best solution found.
11000-11200	Subroutine which updates the current solution.
20000-20200	Print the best solution found by the end of the search.

```
5 RANDOMIZE
10 REM Program for assigning variables for RSM and finding the number of
20 REM confounded terms.
30 DIM V(40): REM These are the values for the cross terms.
50 DIM TF$(40): REM these are the important cross terms.
60 DIM A$(10): REM these are the factors.
70 DIM P$(10): REM these are the best solution found.
80 DIM C$(10): REM these are the current solution.
90 TEMP = 5: REM the initial temperature.
100 REM Initial settings for 10 variables to the 10 factors
110 A$(1) = "5": A$(2) = "9": A$(3) = "3":
120 A$(4) = "8": A$(5) = "1": A$(6) = "0"
130 A$(7) = "4": A$(8) = "2": A$(9) = "7": TC = 999
140 A$(10) = "6": TB = 999: REM TB is the best T value to date
150 FOR i = 1 TO 10
160 P$(i) = A$(i)
170 NEXT i
200 REM Defining the interaction terms TF$
210 REM aa=1, ag=2, ga=3 oca=4 eair=5 barcap=6 fswp=7 [illegible] nt=9 bai=0
220 TF$(1) = "49": TF$(2) = "94"
230 TF$(3) = "57": TF$(4) = "75"
240 TF$(5) = "89": TF$(6) = "98"
250 TF$(7) = "14": TF$(8) = "41"
260 TF$(9) = "15": TF$(10) = "51"
270 TF$(11) = "16": TF$(12) = "61"
280 TF$(13) = "17": TF$(14) = "71"
290 TF$(15) = "18": TF$(16) = "81"
300 TF$(17) = "19": TF$(18) = "91"
310 TF$(19) = "10": TF$(20) = "01"
320 TF$(21) = "12": TF$(22) = "21"
330 TF$(23) = "13": TF$(24) = "31"
340 TF$(25) = "24": TF$(26) = "42"
350 TF$(27) = "26": TF$(28) = "62"
360 TF$(29) = "34": TF$(30) = "43"
370 TF$(31) = "37": TF$(32) = "73"
380 TF$(33) = "38": TF$(34) = "83"
390 TF$(35) = "93": TF$(36) = "39":
400 TF$(37) = "03": TF$(38) = "30": REM remove 03 and 30 and 0 value sol exists
405 TF$(39) = "**": TF$(40) = "**"
```

```
410 REM Now to enter the values for the cross terms.
420 V(1) = 100: V(2) = V(1)
421 V(3) = 99: V(4) = V(3)
422 V(5) = 98: V(6) = V(5)
423 V(7) = 95: V(8) = V(7)
424 V(9) = 80: V(10) = V(9)
425 V(11) = 58: V(12) = V(11)
426 V(13) = 28: V(14) = V(13)
427 V(15) = 25: V(16) = V(15)
428 V(17) = 23: V(18) = V(17)
429 V(19) = 21: V(20) = V(19)
430 V(21) = 20: V(22) = V(21)
431 V(23) = 12: V(24) = V(23)
432 V(25) = 8: V(26) = V(25)
433 V(27) = 7: V(28) = V(27)
434 V(29) = 5: V(30) = V(29)
435 V(31) = 4: V(32) = V(31)
436 V(33) = 3: V(34) = V(33)
437 V(35) = 2: V(36) = V(15)
438 V(37) = 2: V(38) = V(17)
439 V(39) = 1: V(40) = V(19)
1000 REM now to count the confounding. each 100 is a constraint
1010 T = 0: x1 = 0
1100 FOR i = 1 TO 38
1110 IF A$(1) + A$(2) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1120 IF A$(9) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1160 NEXT i
1170 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1180 x1 = 0: Vtemp = 0
1200 FOR i = 1 TO 38
1210 IF A$(1) + A$(3) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1220 IF A$(8) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1260 NEXT i
1270 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1280 x1 = 0: Vtemp = 0
1300 FOR i = 1 TO 38
1310 IF A$(1) + A$(4) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1320 IF A$(7) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1360 NEXT i
1370 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1380 x1 = 0: Vtemp = 0
1400 FOR i = 1 TO 38
1410 IF A$(1) + A$(5) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1420 IF A$(6) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1460 NEXT i
1470 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1480 x1 = 0: Vtemp = 0
1500 FOR i = 1 TO 38
1510 IF A$(1) + A$(6) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1520 IF A$(5) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1560 NEXT i
1570 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1580 x1 = 0: Vtemp = 0
```

```
1600 FOR i = 1 TO 38
1610 IF A$(1) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1620 IF A$(4) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1660 NEXT i
1670 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1680 x1 = 0: Vtemp = 0
1700 FOR i = 1 TO 40
1710 IF A$(1) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1720 IF A$(3) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1760 NEXT i
1770 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1780 x1 = 0: Vtemp = 0
1800 FOR i = 1 TO 38
1810 IF A$(1) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1820 IF A$(2) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1860 NEXT i
1870 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1880 x1 = 0: Vtemp = 0
1900 FOR i = 1 TO 38
1910 IF A$(1) + A$(10) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1920 IF A$(2) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1930 IF A$(3) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1940 IF A$(4) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1950 IF A$(5) + A$(6) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
1960 NEXT i
1970 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
1980 x1 = 0: Vtemp = 0
2100 FOR i = 1 TO 38
2110 IF A$(2) + A$(3) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2120 IF A$(8) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2160 NEXT i
2170 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2180 x1 = 0: Vtemp = 0
2200 FOR i = 1 TO 38
2210 IF A$(2) + A$(4) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2220 IF A$(7) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2260 NEXT i
2270 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2280 x1 = 0: Vtemp = 0
2300 FOR i = 1 TO 38
2310 IF A$(2) + A$(5) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2320 IF A$(6) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2360 NEXT i
2370 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2380 x1 = 0: Vtemp = 0
2400 FOR i = 1 TO 38
2410 IF A$(2) + A$(6) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2420 IF A$(5) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2460 NEXT i
2470 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2480 x1 = 0: Vtemp = 0
2500 FOR i = 1 TO 38
2510 IF A$(2) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
```

```
2520 IF A$(4) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2560 NEXT i
2570 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2580 x1 = 0: Vtemp = 0
2600 FOR i = 1 TO 38
2610 IF A$(2) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2620 IF A$(3) + A$(9) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2660 NEXT i
2670 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2680 x1 = 0: Vtemp = 0
2700 FOR i = 1 TO 38
2710 IF A$(3) + A$(4) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2720 IF A$(7) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2760 NEXT i
2770 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2780 x1 = 0: Vtemp = 0
2800 FOR i = 1 TO 38
2810 IF A$(3) + A$(5) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2820 IF A$(6) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2860 NEXT i
2870 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2880 x1 = 0: Vtemp = 0
2900 FOR i = 1 TO 38
2910 IF A$(3) + A$(6) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2920 IF A$(5) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
2960 NEXT i
2970 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
2980 x1 = 0: Vtemp = 0
3100 FOR i = 1 TO 38
3110 IF A$(3) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3120 IF A$(4) + A$(8) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3160 NEXT i
3170 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
3180 x1 = 0: Vtemp = 0
3200 FOR i = 1 TO 38
3210 IF A$(4) + A$(5) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3220 IF A$(6) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3260 NEXT i
3270 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
3280 x1 = 0: Vtemp = 0
3300 FOR i = 1 TO 38
3310 IF A$(4) + A$(6) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3320 IF A$(5) + A$(7) = TF$(i) THEN x1 = x1 + 1: Vtemp = Vtemp + V(i)
3360 NEXT i
3370 IF x1 > 1 THEN T = T + Vtemp: REM T = total value of confounding interactions
3380 x1 = 0: Vtemp = 0
3400 REM countering and feedback to user
3410 counter = counter + 1: PRINT TC;
3420 IF counter = 30 THEN TEMP = TEMP * .5: PRINT "TEMP= "; TEMP: counter = 0
3440 IF TEMP < .05 THEN GOTO 20000: REM These lines control how long to spend at one temp
3450 REM and how quickly the temperature falls. 3440 controls when to stop.
4000 IF T < TB THEN C = 0: TB = T: GOSUB 10000: REM updating best solution.
4005 IF T = TB THEN GOSUB 10000
```

```
4010 IF T < TC * (TEMP + 1) THEN GOSUB 11000
5000 REM need to generate a new selection of variables.
5010 f1 = INT(RND * 10) + 1
5020 f2 = INT(RND * 10) + 1
5030 IF f1 = f2 THEN GOTO 5010
5040 FOR j = 1 TO 10
5050 A$(j) = C$(j)
5060 NEXT j
5070 REM resets a$ to current C$ values
5200 REM swap two variable assignments between factors a$(f1) and a$(f2)
5210 REM temp$ is a temporary variable
5220 TEMP$ = A$(f2)
5230 A$(f2) = A$(f1)
5240 A$(f1) = TEMP$
5232 flips = flips + RND
5245 IF flips < TEMP + .3 THEN GOTO 5000
5347 flips = 0
5250 REM redo calculations
5260 GOTO 1000
10000 REM update best solution
10010 FOR i = 1 TO 10
10020 P$(i) = A$(i)
10025 IF C < 10 THEN IF TB < 30 THEN PRINT A$(i),
10030 REM PRINT A$(i),
10040 NEXT i
10045 IF C < 10 THEN IF TB < 30 THEN PRINT "value of interactions is "; TB: C = C + 1
10050 REM "The value of confounded interactions is "; TB
10100 IF TB < 2 THEN END
10110 RETURN
11000 REM updating the current solution
11100 FOR i = 1 TO 10
11110 C$(i) = A$(i)
11120 NEXT i
11130 TC = T
11140 RETURN
20000 REM routine to end program
20100 PRINT : "Best solution found:"
20110 FOR i = 1 TO 10
20120 PRINT "Factor "; i; "should be assigned variable "; P$(i)
20130 NEXT i
20140 PRINT "The value of the interactions is "; TB
20150 END
```

APPENDIX C

The design for the baseline experiment and the resulting data are presented in this appendix. The design for the sensitivity analysis and the resulting data used in chapter four are also presented.

run	OCA	AIRESC	SSUP	SJAM	BCAP	FSWP	CAS	INT	BAI	FLOT	Grd Str	Air Atr
1	-1.	-1.	-1.	-1.	-1.	-1.	-1.	-1.	1.	1098.0	44.6	14.6
2	1.	-1.	-1.	-1.	1.	-1.	1.	1.	-1.	43.2	70.1	10.7
3	-1.	1.	-1.	-1.	1.	1.	-1.	1.	-1.	659.8	53.9	23.8
4	1.	1.	-1.	-1.	-1.	1.	1.	-1.	1.	81.3	62.5	12.4
5	-1.	-1.	1.	-1.	1.	1.	1.	-1.	-1.	102.5	62.5	10.5
6	1.	-1.	1.	-1.	-1.	1.	-1.	1.	1.	662.7	51.5	39.0
7	-1.	1.	1.	-1.	-1.	-1.	1.	1.	1.	45.3	69.6	5.8
8	1.	1.	1.	-1.	1.	-1.	-1.	-1.	-1.	1056.7	48.4	20.0
9	-1.	-1.	-1.	1.	-1.	1.	1.	1.	-1.	44.0	69.1	9.0
10	1.	-1.	-1.	1.	1.	1.	-1.	-1.	1.	1015.5	47.2	31.6
11	-1.	1.	-1.	1.	1.	-1.	1.	-1.	1.	105.8	62.1	8.1
12	1.	1.	-1.	1.	-1.	-1.	-1.	1.	-1.	691.3	52.6	26.2
13	-1.	-1.	1.	1.	1.	-1.	-1.	1.	1.	750.3	48.4	33.5
14	1.	-1.	1.	1.	-1.	-1.	1.	-1.	-1.	100.0	64.6	10.6
15	-1.	1.	1.	1.	-1.	1.	-1.	-1.	-1.	1073.0	44.4	6.8
16	1.	1.	1.	1.	1.	1.	1.	1.	1.	42.5	69.5	9.1

The high and low levels for each mission are:

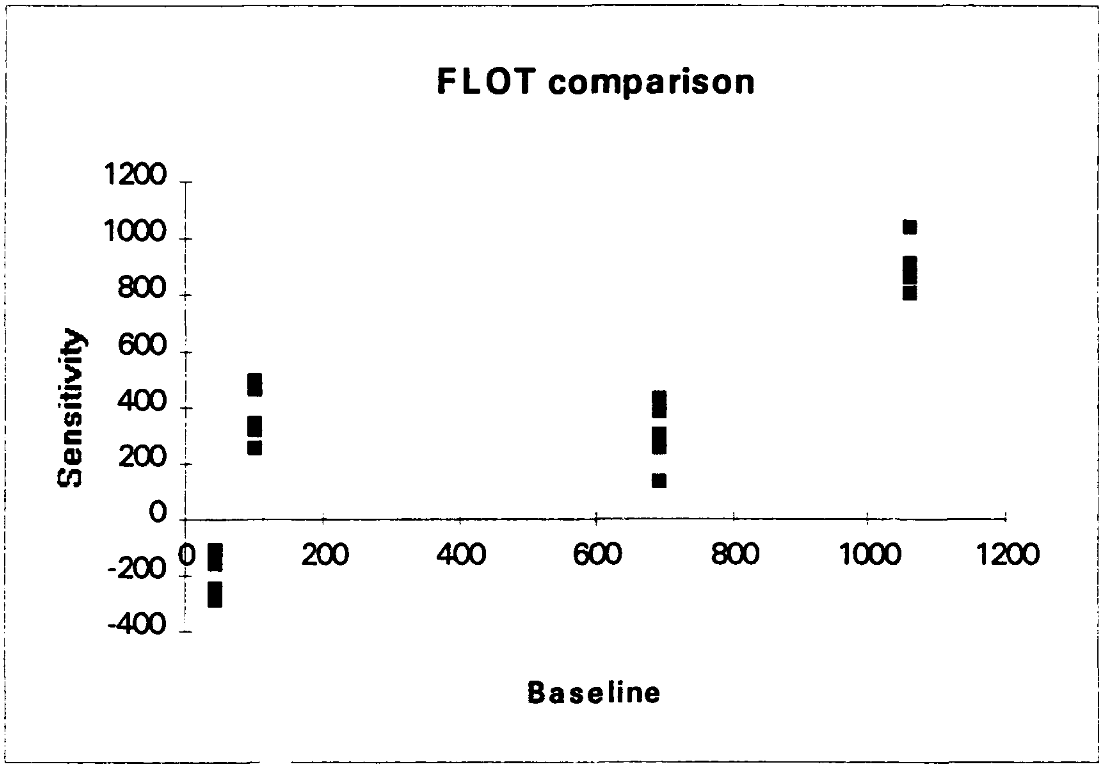
MISSION	HIGH	LOW	MISSION	HIGH	LOW
FSWP	72	12	BAI	98	0
AIRESC	72	12	SJAM	24	8
BARCAP	72	12	SSUP	30	0
INT	96	0	OCA	72	12
CAS	216	0			

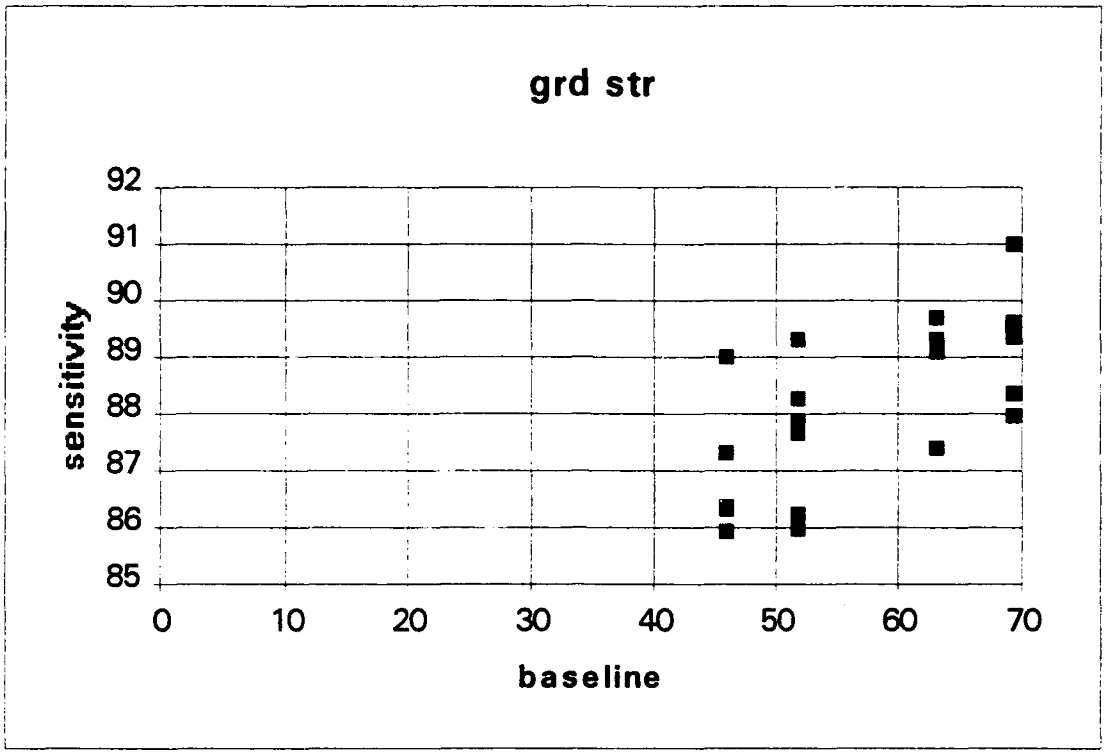
The data from chapter four:

run	AAPk	FSWP	AIRESC	CAS	SAPk	BCAP	INT	BAI	OCA	FLOT(ln(km))	Grd Str	Air Atr
1	-1.	-1.	-1.	-1.	-1.	1.	1.	1.	1.	6.456217	88.2	34.4
2	1.	-1.	-1.	-1.	-1.	1.	-1.	-1.	-1.	6.969432	85.0	16.1
3	-1.	1.	-1.	-1.	-1.	-1.	1.	-1.	-1.	6.559227	86.0	22.8
4	1.	1.	-1.	-1.	-1.	-1.	-1.	1.	1.	6.953589	85.2	38.5
5	-1.	-1.	1.	-1.	-1.	-1.	-1.	1.	-1.	6.995636	84.5	10.8
6	1.	-1.	1.	-1.	-1.	-1.	1.	-1.	1.	6.533760	85.8	30.0
7	-1.	1.	1.	-1.	-1.	1.	-1.	-1.	1.	6.967423	85.4	21.1
8	1.	1.	1.	-1.	-1.	1.	1.	1.	-1.	6.953225	85.9	25.5
9	-1.	-1.	-1.	1.	-1.	-1.	-1.	-1.	1.	6.950029	85.5	21.4
10	1.	-1.	-1.	1.	-1.	-1.	1.	1.	-1.	3.848006	87.9	13.1
11	-1.	1.	-1.	1.	-1.	1.	-1.	1.	-1.	4.558096	86.4	10.0
12	1.	1.	-1.	1.	-1.	1.	1.	-1.	1.	3.858519	88.4	18.7
13	-1.	-1.	1.	1.	-1.	1.	1.	-1.	-1.	3.964299	88.8	8.7
14	1.	-1.	1.	1.	-1.	1.	-1.	1.	1.	4.604133	87.2	18.4
15	-1.	1.	1.	1.	-1.	-1.	1.	1.	1.	3.709229	89.0	8.9
16	1.	1.	1.	1.	-1.	-1.	-1.	-1.	-1.	4.615815	86.5	8.4
17	-1.	-1.	-1.	-1.	1.	-1.	-1.	-1.	-1.	6.975149	84.6	11.2
18	1.	-1.	-1.	-1.	1.	-1.	1.	1.	1.	3.784849	88.2	10.1
19	-1.	1.	-1.	-1.	1.	1.	-1.	1.	1.	6.937167	85.8	42.8
20	1.	1.	-1.	-1.	1.	1.	1.	-1.	-1.	6.512805	86.8	48.7
21	-1.	-1.	1.	-1.	1.	1.	1.	-1.	1.	6.486638	87.5	33.9
22	1.	-1.	1.	-1.	1.	1.	-1.	1.	-1.	6.927324	85.6	34.5
23	-1.	1.	1.	-1.	1.	-1.	1.	1.	-1.	6.562940	86.0	37.9
24	1.	1.	1.	-1.	1.	-1.	-1.	-1.	1.	6.997021	85.0	40.0
25	-1.	-1.	-1.	1.	1.	1.	1.	1.	-1.	3.804438	88.1	10.1
26	1.	-1.	-1.	1.	1.	1.	-1.	-1.	1.	4.436076	87.8	18.5
27	-1.	1.	-1.	1.	1.	-1.	1.	-1.	1.	3.926964	88.2	14.3
28	1.	1.	-1.	1.	1.	-1.	-1.	1.	-1.	5.017548	86.1	17.7
29	-1.	-1.	1.	1.	1.	-1.	-1.	1.	1.	4.548363	87.0	6.3
30	1.	-1.	1.	1.	1.	-1.	1.	-1.	-1.	3.887047	88.8	8.1
31	-1.	1.	1.	1.	1.	1.	-1.	-1.	-1.	4.511347	87.0	12.1
32	1.	1.	1.	1.	1.	1.	1.	1.	1.	3.710947	88.0	21.6

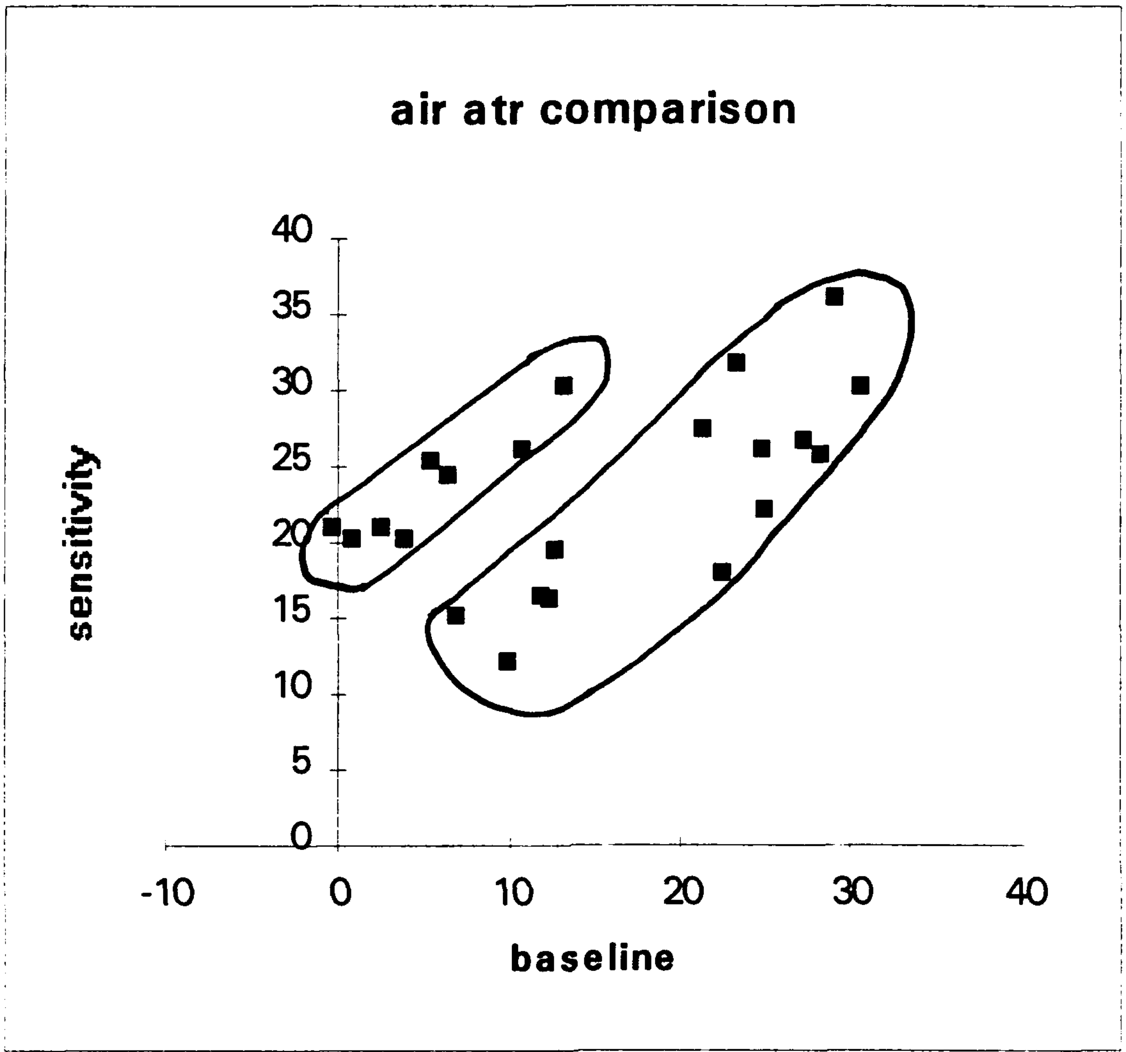
APPENDIX D

This appendix contains the plots used to compare the baseline models to the sensitivity models. The sensitivity models had the SAPk and AAPk terms equal to -1. The FLOT comparison shows a reasonable correlation when the predicted FLOT movement exceeds 500km. The sensitivity model predicted negative FLOT values for design points that the baseline model predicted a FLOT movement of approximately 50 km.





The ground strength comparison showed no unusual features.



The air attrition comparison showed groupings which appeared to be correlated. The upper left data points all had the maximum number of CAS missions. Once again, CAS seems to have a significant impact on the air attrition process. Further investigation is warranted on the relationship between air attrition and CAS missions.

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13. ABSTRACT *(Maximum 200 words)*

To effectively compare different sets of available aircraft, it is necessary to find the apportionment that maximizes the effectiveness of each set of aircraft. This research effort uses an unclassified TAC THUNDER scenario to show how an analyst can use response surface methodology (RSM) techniques to estimate the relationship between aircraft apportionment and campaign outcomes. The optimal apportionments for one phase of a hypothetical conflict was found by using a steepest-gradient search of the constrained response surfaces. The results are illustrated by illuminating charts showing the various relationships between the number of aircraft in the theater and measures of effectiveness such as FLOT movement and attrition. The results show close air support and interdiction missions to be highly effective in this phase of the scenario. The effect of the enemy's air-to-air probability of kill (AAPk) and surface-to-air probability of kill (SAPk) on the MOEs provides insight into the sensitivity of the solution in terms of these operational uncertainty parameters. This type of sensitivity analysis also provides useful information on how the TAC THUNDER algorithms interact to simulate the combat environment.

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